Appraising the "Merger Price" Appraisal Rule[*]

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Abstract

This paper develops an analytic framework combining agency costs, auction design and shareholder voting to study how best to measure "fair value" for dissident shareholders in a post-merger appraisal proceeding. Our inquiry spotlights an approach recently embraced by some courts that benchmarks fair value against the merger price itself, at least in certain situations. As a general matter, the "Merger Price" (MP) rule tends to depress both acquisition prices and target shareholders' expected welfare relative to both the optimal appraisal policy and several other plausible alternatives. In fact, we demonstrate that the MP rule is strategically equivalent to *nullifying* appraisal rights altogether. Although the MP rule may be warranted in certain circumstances, our analysis suggests that such conditions are unlikely to be widespread and, consequently, the rule should be employed with caution. Our framework also helps explain why a healthy majority of litigated appraisal cases using conventional fair-value measures result in valuation assessments exceeding the deal price—an equilibrium phenomenon that is an artifact of rational, strategic behavior (and not necessarily an institutional deficiency, as some assert). Finally, our analysis facilitates better understanding of the strategic and efficiency implications of recent reforms allowing "medium-form" mergers, as well as an assortment of (colorfully named) appraisal-related practices, such as blow provisions, drag-alongs, and "naked no-vote" fees.

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Introduction

In mergers and acquisitions law, the appraisal right affords target-company shareholders an option of eschewing the terms of an acquisition in favor of receiving a judicially determined cash valuation for their shares. All states provide this statutory option in some form for many—though not all—transactions.[1] In eligible cases, appraisal gives dissenting shareholders a potentially powerful tool to counter deal terms that they believe to be inadequate or under-compensatory. Although public company targets have historically faced appraisal actions only rarely, the procedure has grown significantly more popular and prevalent in recent years.[2]

Appraisal proceedings are far less popular, by contrast, among judges who preside over them. A particularly vexing challenge facing courts in such matters is the task of distilling metaphorical mountains of financial and technical data into a singular equitable determination of "fair value."[3] The judge usually cannot dodge this responsibility on procedural grounds, cannot hand off the job to a jury, and cannot take refuge in traditional jurisprudential heuristics—such as evidentiary burdens of proof.[4] Rather, the typical appraisal proceeding allocates *no explicit burden of proof* and *requires* the court to deliver a single number at the end of the process. Testimony in such proceedings typically adds little solace, dominated by prolix technical reports from litigant-retained experts whose valuation opinions can diverge by as much as an order of magnitude. Especially for judges who are uncomfortable with the intricacies of asset pricing, fair valuation can be a formidable beast to wrangle.[5]

[1] Delaware's statute is somewhat narrow relative to others, limited to statutory merger transactions. See DGCL §262. And even there, when the target's stock is publicly traded, Delaware limits appraisal eligibility to acquisitions involving mandatory non-stock (including cash) consideration, as well squeeze-out mergers under §§253 and 251(h). See DGCL §262(b). Many states outside Delaware fashion their corporate statutes after the Model Business Corporations Act, which grants more liberal appraisal rights following (1) mergers, (2) share exchanges, (3) dispositions of assets, (4) amendments to the articles, and (5) conversion or domestication. See RMBCA § 13.2.

[2] See, e.g., Korsmo and Myers (2015 and 2016), documenting the recent rise of appraisal actions, which grew from affecting 5% of eligible deals in 2003 to over 20% as of 2013, along with a roughly six-fold increase in aggregate monetary claims over the same period. Sophisticated institutional shareholders (including hedge funds) appear to be playing an important role in this recent upsurge.

[3] See, e.g., DGCL §262(h) (requiring the Chancery Court to determine the "fair value" of the shares "exclusive of any element of value arising from the accomplishment of expectation of the merger or consolidation").

[4] In some isolated cases, shareholders have been disqualified for failing to adhere to the formal process for perfecting appraisal rights. See, e.g., In re Appraisal of Dell Inc., 143 A.3d 20, 21-23 (Del. Ch. 2016) (disqualifying dissenting shareholders from seeking appraisal as to shares mistakenly cast for the merger by street-name record holder).

[5] See, e.g., In re Appraisal of Ancestry.com, 2015 WL 399726, at 2 ("this task is made particularly difficult for the bench judge, not simply because his training may not provide a background well-suited to the process, but also because of the way the statute is constructed. A judge in Chancery is the finder of fact, and is frequently charged to make difficult factual determinations that may be without his area of expertise…in reality, the 'burden' falls on the judge to determine fair value, using —all relevant factors").

In several recent appraisal cases,[6] the Delaware Chancery Court has deployed a jurisprudential *verónica*[7] of sorts—invoking a doctrine that sidesteps this valuation challenge substantially (if not altogether). Specifically, the Court has proven increasingly willing to use *the merger price itself* as evidence (and sometimes *the decisive* piece of evidence) of fair value. To date, the "Merger Price" (MP) rule has not been utilized categorically, but instead seems confined largely to settings where the transaction "resulted from a competitive and fair auction, which followed a more-than-adequate sales process and involved broad dissemination of confidential information to a large number of prospective buyers."[8] Nevertheless, even in deals that engage a single bidder in bilateral negotiations, courts increasingly accord the merger price "substantial evidentiary weight" when the transaction "resulted from an arm's-length process between two independent parties, and…no structural impediments existed that might materially distort—the crucible of objective market reality."[9] In fact, several advocates and at least some academic commentators have sought to nudge the MP rule further in this direction, arguing that courts should defer "entirely" to the merger price when it is the product of a reasonable and disinterested process.[10]

The concept underlying the MP rule is easy enough to articulate: it posits that "The Market" delivers the best indication of fair value,[11] so long as the deal price is a product of reasonable arm's-length negotiations. In other words, the MP rule is a natural corollary of the (seemingly intuitive) economic intuition that a fully shopped deal provides adequate pricing protection to target shareholders, and that in such cases market price is a better bellwether of value than a judge's often arbitrary, error-prone, and inaccurate accounting.

Sounds simple, right?

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[6] See, e.g., Merion Capital v. Lender Processing Services, C.A. No. 9320-VCL (Del. Ch. Dec. 16, 2016); Dunmire v. Farmers & Merchants Bancorp, C.A. No. 10589-CB (Del. Ch. 2016); Merion Capital v. BMC Software, C.A. No. 8900-VCG (2015); Huff Fund Inv. P'ship v. CKx, Inc., 2013 WL 5878807 (Del. Ch. Nov. 1, 2013); In re Appraisal of Ancestry.com, 2015 WL 399726 (Del. Ch. Jan. 30, 2015); LongPath Capital LLC v. Ramtron International Corp., C.A. No. 8094-VCP (Del. Ch. June 30, 2015); Merlin Partners LP v. AutoInfo Inc., C.A. No. 8509-VCN (Del. Ch. Apr. 30, 2015); The Union Illinois 1995 Investment Limited Partnership v. Union Financial Group, Ltd., 847 A.2d 340 (Del. Ch. 2004).

[7] *See, e.g.*, Angela Tung, "Them's Bullfighting Words." The Week (March 25, 2015); "Jose Tomas a la Verónica en Sevilla" (available at https://youtu.be/NZD4N7plZNQ).

[8] Dunmire, supra at 20 (quoting Union Illinois 1995 Inv. Ltd. P'ship v. Union Fin. Grp., Ltd., 847 A.2d 340, 358 (Del. Ch. 2004)). In 2010, the Delaware Supreme Court, in rejecting a categorical MP rule, held that the merger price was a permissible consideration in determining fair value. Golden Telecom Inc. v. Global GT LP, 11 A.3d 214 (Del. 2010).

[9] See In Re Ancestry.com, supra note 5 at 35 (quoting Highfields Capital, Ltd. v. AXA Fin., Inc., 939 A.2d 34, at 42 (Del. Ch. 2007).

[10] See, e.g., Bainbridge et al., Brief of Corporate Law and Finance Professors as *Amici* in Support of Reversal, DFC Global Corp. v. Muirfield Value Partners et al. (Del. Sup. Court, Dec. 29, 2016); Appellants Opening Brief, DFC Global Corp. V. Muirfield Value Partners et al., No. 518, 2016 On Appeal from the Court of Chancery of the State of Delaware, Consolidated C.A. No. 10107-CB (Del. Sup. Court, Dec. 13, 2016).

[11] See, e.g., See, e.g., Merion Capital LP v. BMC Software, Inc., 2015 WL 6164771, at *14–16, *18 (Del. Ch. Oct. 21, 2015); LongPath Capital, LLC v. Ramtron Int'l Corp., 2015 WL 4540443, at *20–24 (Del. Ch. June 30, 2015).

Not so fast. This paper demonstrates that the intuition underlying the MP rule—while sound in certain respects—is less general and more fragile than it first appears. Specifically, we show that the rule is defensible on economic grounds only in relatively narrow set of circumstances that can be demanding, in practice, to meet; and in any event, such circumstances are difficult to diagnose *without* the court going much of the way to value the firm using more conventional measures. Consequently, if the primary benefit of the MP approach is judicial cost savings, the approach can be self-defeating.

Our argument begins by highlighting a critical flaw in the economic logic that purportedly animates the MP rule: The presumption that "The Market" operates separately and independently from the underlying legal environment. On first principles alone, *this presumption is generally false; market outcomes and laws governing markets are fundamentally intertwined.* Markets—particularly robust ones—amalgamate and reflect participants' expectations about the future, related to earnings, costs, new business opportunities, and the like. But healthy markets *also* reflect participants' expectations about the very legal environment in which markets operate.[12] Change that legal environment, and expectations will change; change expectations, and market prices soon follow. It is a fundamental economic misconception, therefore, to presume that a market price—even one produced by a seemingly robust market—is an autonomous oracle of worth, untethered to expectations related to (and affected by) law.

While the interdependency of market price and legal environment transcends many fields of practice,[13] it carries particular bite in the appraisal context: for a court's approach to assessing fair value in appraisal affects not only what dissenting shareholders receive *ex post*, but also how the merger is priced and approved (or not) *ex ante*. Indeed, the outside option of seeking appraisal after a merger can functionally alter shareholders' receptivity to an announced deal, effectively committing them to a "reserve price" of sorts for the sale, at an amount tied to the anticipated appraisal remedy. Under plausible conditions, this *de facto* reserve price can protect shareholders' interests more ably than either a shareholder approval requirement, or reliance on management's incentives to design a profit maximizing auction. Sophisticated buyers, moreover, anticipate this effect, and may well modify their bids in response, adjusting them upward to meet (or get close to) the appraisal reserve price, secure shareholder approval, and preempt widespread appraisal litigation. To the extent that appraisal value is pegged against independent factors (and *not* the merger price), a plausibly designed appraisal remedy can enhance value for *all* shareholders—even those who do not seek appraisal.

Under the MP rule, by contrast, this reserve-price effect collapses under its own weight. Indeed, the MP rule dictates that the value of shareholders' appraisal right floats up and down mechanically with the winning bid, regardless of the bid's evident adequacy under objective measures. Opting for appraisal, therefore, can never yield a dissenter any upside over the terms of the merger (and may introduce a downside in the form of legal costs). Consequently, prospective buyers need not fear that the winning bid will prove

[12] See, e.g., Cornell (1990).
[13] For more on the circularity that ensues when market price is used to determine a legal outcome—even as the anticipated legal outcome simultaneously determines market price—see Talley (2006).

inadequate relative to the outside option of appraisal: for *the winning bid is the outside option.* Put simply, the MP rule functionally nullifies the appraisal right, and whatever value enhancing implications the reserve-price effect portends. So long as there exists some plausible alternative appraisal remedy that enhances shareholders' welfare *ex ante*—even if modestly—the MP rule cannot be optimal.

To demonstrate our claims, we analyze a canonical auction framework from economics, involving a group of arm's-length buyers who may bid on a target firm. Our framework incorporates several features that are specific to the corporate acquisition context, including agency costs associated with the deal team's auction-design incentives, a shareholder voting requirement to close a signed deal, and a post-transaction appraisal remedy for dissenters. Using this framework, we compare equilibria under what we call "conventional" appraisal valuation approaches (where information unrelated to the winning bid is used to gauge value[14]) to the MP rule (where appraisal value is pegged at the winning bid). Holding the number of bidders fixed, we show that the MP rule *never* generates a higher price than plausible conventional approaches, and more typically leads to a strictly lower price. Our results, moreover, extend beyond mere price, holding implications for expected shareholder welfare too: whenever any plausible conventional approach to valuation enhances *ex ante* shareholder value relative to no appraisal rights, the MP approach must necessarily be inferior. Although we identify some circumstances where the MP rule could conceivably be one of several optimal alternatives, these conditions appear difficult to satisfy in practice. Our analysis therefore counsels that the MP rule should be deployed—if at all—with caution.

Beyond this core contribution, our inquiry sheds light on a variety of debates among commentators and academics related to appraisal. Most notably, our model predicts that under a conventional appraisal rule, shareholders will—in equilibrium—seek appraisal only rarely, and typically only for mergers that offer relatively meager premiums. Moreover, in those instances where appraisal is sought, fair-value assessments will tend overwhelmingly to be skewed *well above* the deal price. Each of these predictions appears to have solid empirical support.[15] Nevertheless, several proponents of the MP rule have pointed to the upwards skew of appraisal awards relative to deal price as evidence of dysfunction in the process. Our analysis parts company with that conclusion: the upward skew we predict is a simple artifact of rational, strategic decision making. When target shareholders expect the appraisal valuation to be *lower* than the merger consideration, they will simply decline to seek appraisal (even if they oppose the acquisition). We would therefore expect to see a qualitatively similar upward skew on appraisal awards regardless of whether the fair-value measure is set too high, too low, or just right by objective criteria.[16]

[14] Discounted Cash Flow (DCF) analysis is the predominant conventional approach today, but the class of conventional methods includes all other methods that are independent of Merger Price, such as comparable companies/transactions approaches and the old Delaware "block" method for valuation. See Allen (2002).
[15] See, e.g., Korsmo and Myers (2015 and 2016).
[16] That said, our analysis also suggests that it may be equally premature to celebrate recent increases in appraisal incidence. Within our framework, the equilibria that involve appraisal actions tend to yield low expected payoffs for shareholders compared to other attainable equilibria involving less litigation.

Our framework additionally illuminates the strategic and efficiency implications of several institutional devices that go beyond the MP rule, but which fundamentally bear on appraisal. For example, a popular deal structure for public-company targets in Delaware—and one where appraisal is usually available—involves a negotiated tender offer, followed immediately by an involuntary "squeeze out" merger of non-tendering shareholders at the same price. Historically, in order to close the deal quickly, it was functionally necessary for at least 90-percent of target's shareholders to tender into the first stage.[17] In 2013, however, Delaware amended its statutes to allow an alternative form of two-step merger, wherein the first step need only secure a bare 50-percent threshold before an accelerated squeeze out can commence.[18] A central result of our analysis is that the MP rule can be optimal when the merger is conditioned on a strong super-majority approval of shareholders.[19] This insight suggests that courts might similarly condition their valuation approach on the shareholder mandate sought: traditional two-step deals requiring 90-percent support would receive the MP rule, while "medium-form" deals requiring a mere 50-percent would fall under more conventional approaches (such as DCF).

Moreover, our analysis also facilitates the evaluation of several appraisal-related contractual terms. For example, "drag-along" provisions oblige shareholders to vote in favor of a merger when a sufficient fraction of shareholders favors the acquisition. "Naked no vote" terms require the target to pay a termination fee to the buyer should the deal be vetoed by shareholders. "Blow" provisions condition the buyer's duty to close a merger on a maximal threshold of shareholders seeking appraisal (frequently in the 10-20 percent range). Each of these devices can play multiple roles in our model of (a) reallocating surplus between the winning bidder, supporting shareholders and dissenting shareholders; (b) altering the sets of shareholders disposed to support or oppose the merger; and (c) changing the incentives for optimal auction design. Our analysis suggests that drag-alongs and naked no-vote provisions tend to dampen deal prices and target shareholder welfare, negating many of the beneficial attributes of appraisal. Blow provisions, in contrast, have more complex effects: Although a blow clearly rations appraisal's availability to a relatively select fraction of target shareholders, it simultaneously signals an implicit supermajority condition for the deal's consummation. As noted above, such supermajority conditions can often accomplish the same purpose as an optimal appraisal rule, pushing merger prices and shareholder welfare upwards. (In fact, in certain circumstances the MP rule might even be optimal in the presence of an effective blow.) Although we comment briefly on each of these effects below, our analytic framework provides a promising platform from which to analyze these contractual institutions in more granular detail.

Several important caveats to our core argument warrant elaboration before proceeding. First, although the price- and welfare-dampening attributes of the MP rule hold for auctions of any size, the quantitative magnitudes of these effects attenuate as the number of bidders grows. That is, the MP rule visits progressively smaller discounts on

[17] See DGCL § 253.
[18] See DGCL § 251(h).
[19] See Proposition 7, infra.

target share value as the bidder population expands. (In the limit, as the bidder population grows arbitrarily large, the discount from the MP rule approaches zero.) Consequently, when the number of bidders is *endogenous* to the seller's efforts to shop the deal, the appraisal rule can represent a compelling incentive device. For example, if the MP rule were available only after large and robust auctions, the seller's deal team may have a much stronger incentive to design an effective auction. In such settings, shareholder welfare may well be higher when (a) numerous bidders participate but the MP rule nullifies appraisal rights, than when (b) relatively fewer bidders bid in the shadow of a bona fide appraisal right. Thus, were the MP rule strictly limited to "many-bidder" settings, its downside would be relatively modest (and its upside intriguing).[20]

Second, as noted above, a standard knock against conventional approaches to fair value (such as DCF) is that they are prone to measurement error when utilized by judges who are not financial experts. Our analysis allows for this possibility. In fact, virtually all our arguments remain intact even when appraisal proceedings are subject to potentially severe judicial inaccuracy, *so long as* courts remain unbiased overall – that is, if *the distribution* of appraisal valuations remains stable and roughly predictable (even if subject to uncertainty in each individual case). The reason is simple: Much of the reserve-price benefit of appraisal inures to shareholders by enhancing buyers' willingness to pay higher premiums *ex ante*, so as to win affirmative votes and avoid appraisal. In the presence of judicial error, both the buyer's and the prospective dissenters' calculi change, replacing a *predictable* appraisal value with its *expected* value. But so long as error-prone courts remain unbiased overall, the substitution of expected values will have only marginal effects, and bidding and dissenting behaviors remain largely unchanged.[21]

Third, it is important to recognize that appraisal is one of *several* alternative mechanisms that introduce an implicit reserve price in a company auction. Another is shareholder voting. Our analysis engages this possibility explicitly, demonstrating that the required approval of target shareholders implicitly provides an alternative reserve price: if the "pivotal" (usually the median) shareholder views the merger price as insufficiently attractive relative to her valuation, the transaction will not be approved and the acquisition will fail. The standard requirement of a target shareholder vote, therefore, already provides a type of reserve price pegged at the swing voter's valuation. That said, our model shows that shareholder voting need not always substitute for a meaningful appraisal doctrine in at least two respects. First, voting outcomes are pegged against the preferences of the *pivotal* voter, whose preferences need not coincide with overall value

[20] As noted above, however, there has been a recent push by courts and commentators to expand the doctrine beyond these bounds, according the merger price substantial (or even exclusive) weight even when the deal is the product of bilateral one-on-one negotiations. *See* text accompanying notes 9-10, *supra.*

[21] The argument in the text presumes risk neutrality of the buyer and target shareholders (ensuring the strategic equivalence of a fixed appraisal value and a mean-preserving stochastic valuation around the same fixed value). When parties are risk averse, however, our results will change, but could grow *even stronger*. With risk aversion, judicial inaccuracy imposes additional litigation risk on the winning buyer, making the reserve-price effect even more potent in amplifying bids. That said, the added risk also implicates dissenters, giving the parties a strong incentive to settle. Assuming such settlements occurred at approximately the expected appraisal outcome, judicial inaccuracy would add or subtract little from our baseline argument.

maximization. And second, voting can be an inherently unpredictable check on the sale process, since it can generate multiple equilibria that introduce coordination problems for shareholders.[22] Appraisal rights, in contrast, pose no such challenges.

Relatedly, the potential for multiple equilibria in voting outcomes can have distributional consequences for target shareholders contemplating appraisal. Although we show that conventional appraisal approaches generally dominate the MP rule for *all* shareholders, the benefits of appraisal need not always be evenly distributed. In some equilibria, the rising tide of appraisal lifts all boats, enhancing the payoffs of every shareholder *pro-rata*. In other equilibria, however, the optimal appraisal award will tend to be more generous than the equilibrium merger price, forcing target shareholders to divide into two groups: (i) those who seek (the more lucrative) appraisal; and (ii) those who remain part of the majority voting to support the deal (disqualifying them from appraisal). Ironically, shareholders tend to fare better collectively when they cannot easily coordinate as to who fall into groups (i) and (ii). With greater concentration of ownership among sophisticated hedge funds and institutional investors, however, coordination becomes easier, a fact that may often leave retail investors with the short end of the appraisal stick.[23]

Finally, we focus here on an *economic* account of appraisal, assessing how different valuation approaches fare in enhancing target shareholder welfare (or in some cases, social efficiency[24]). For this approach to have practical traction, one must also presuppose (a) that economic considerations "matter" for appraisal jurisprudence, and (b) that the judicial outcome is not overdetermined by other factors (such as rigid precedent, statutory inflexibility or historical path dependence). As to the first presupposition, no one today seriously challenges the utility of economics in clarifying unsettled issues of corporate law (even if disagreement remains about what *other* considerations deserve equal billing). Indeed, many commentators argue that appraisal law *in particular* should embrace the tenets of financial economics as a central normative commitment.[25] As to the second presupposition, it seems unlikely that non-economic factors pre-ordain the outcome of most modern appraisal cases. Although appraisal statutes certainly contain some hard-and-fast imperatives, they are conspicuous both in what they leave unattended (e.g., how to compute fair value) and in their uniquely tortured linguistic indeterminacy.[26] The common law interpretations that have sprung up around appraisal rights, moreover, appear analogously pliant: In the last half century alone, courts have invented (and then reinvented) major components of appraisal time and again, seemingly undaunted by inelastic precedent or statutory compulsion. These dalliances include prescribing—*inter*

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[22] Indeed, as we show below, shareholder voting can introduce multiple equilibria that cannot always be eliminated, even with standard "refinement" assumptions. See Section II(D), *infra.*

[23] Even here, however, retail investors still fare better than under the MP rule. *See* Section III(A), *infra.*

[24] While we focus on maximizing target shareholder value, our framework can also take on broader aims (such as maximizing the sum of the target shareholders' and buyers' payoffs). See Section III(A), *infra*.

[25] See, e.g., Hamermesh & Wachter (2007) at 47-48; Allen (2003) at 552-3.

[26] For instance, the "market-out" exception to appraisal – and the various exceptions to the exception – under DGCL § 262(b) are widely considered to be a poster child for philological nihilism. Accord Thompson (1995), at 30 (characterizing § 262(b) as embodying "the kind of double negative that should make any legislator's grammar teacher cringe").

alia—what approaches are permissible for fair valuation,[27] what elements of value comprise excludable "merger synergies" in an appraisal,[28] whether to adjust fair value for implicit minority discounts,[29] whether mixed cash/stock deals trigger appraisal rights,[30] whether beneficial owners purchasing after the record date are eligible to seek appraisal,[31] and whether late-purchasing owners must demonstrate that their specific shares were not voted in favor of the merger.[32] Even the underlying *policy rationale* for appraisal appears to be a contingent product of apocryphal provenance and peripatetic evolution[33]—one that appears to be unfolding even still. If anything, the role of post-merger appraisal has grown *more* crucial as courts have progressively narrowed other avenues for challenging mergers.[34] If the recent surge in appraisal activity heralds a "transformation" that invites us to re-imagine appraisal's normative commitments—as several now assert[35]—then economic analysis clearly warrants a seat at the table.

The remainder of this paper proceeds as follows. Section I lays out the fundamental framework we study, combining auction design, shareholder governance, agency costs, and legal appraisal rights. Section II derives equilibria of the model for various appraisal rules. We show that voting and appraisal can interact in significant ways, with appraisal plausibly inducing strategic voting among shareholders. We also derive our central result that the MP rule is usually undesirable for target shareholders. Section III considers a variety of extensions to our core model, including characterizing an optimal valuation measure for fair value. There we show that while the MP rule might, under the right circumstances, be one of many other optimal regimes, those circumstances seem implausible in most circumstances. The last section concludes. The appendix contains two parts. Part A briefly reviews the basics of auction theory and optimal auction design, i.e., the optimal reserve price. Part B contains all the proofs.

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[27] Weinberger v. UOP, Inc., 457 A.2d 701 (Del. 1983) (embracing DCF approaches as a preferred – but not mandatory – alternative to the legacy "block" method of valuation).
[28] Cede & Co. v. Technicolor, Inc., 542 A.2d 1182, 1187 n.8. (Del. 1988).
[29] Cavalier Oil Corp. v. Harnett, Nos. 7959, 1988 WL 15816, at *22-23 (Del. Ch. Feb. 2, 1988), aff'd, 564 A.2d 1137 (Del. 1989) (disallowing the implicit minority discount in a DCF valuation). See also Hamermesh & Wachter (2007).
[30] La. Mun. Police Ees' Ret. System v. Crawford, 918 A.2d 1172 (Del. Ch. 2007).
[31] In re Appraisal of Transkaryotic Therapies, Inc., No. Civ.A. 1554-CC, 2007 WL 1378345, at *1 (Del. Ch. May 2, 2007).
[32] See, e.g., In re Appraisal of Ancestry.com, supra note 5; In re Appraisal of Transkaryotic Therapies, Inc., No. Civ.A. 1554-CC, 2007 WL 1378345, at *1 (Del. Ch. May 2, 2007).
[33] It is commonly asserted that appraisal statutes were originally intended as liquidity-preserving substitutes for shareholder veto rights upon the elimination of unanimity voting rules for acquisitions in the early 1900s. See, e.g., Thompson (1995) at 3-4. The "fit" between the timing of appraisal's introduction and the elimination of unanimity mandates, however, casts significant doubt on that common narrative. *See, id. at* 14-15. Moreover, the significant heterogeneity in statutes – both across jurisdictions and over time – appears inconsistent with a single, monolithic, consensus purpose. Levmore & Kanda (1985) at 431-2. Regardless of appraisal's original intent, most believe that the liquidity preservation goal eventually faded, emancipating appraisal to be deployed for other (heterogeneous) aims. Thompson (1995), for example, reports 11 years' worth of appraisal action data (1984-1994) reflecting a large set of circumstances, ranging from close corporations (13.1%) to public-company minority freeze-outs (35.7%) to cash acquisitions of widely-held public targets (25.0%). Thompson (1995), at 27.
[34] See, e.g., In re Trulia Stockholder Litigation, 129 A.3d 884 (Del. Ch. 2016); Kahn v. M&F Worldwide Corp. 88 A.3d 635 (Del. 2014).
[35] See, e.g., Korsmo & Myers (2016), at 2, 55-56.

I. The Setup

We analyze the potential sale of a corporate entity (or "target" corporation) involving three groups of strategic, risk-neutral players: incumbent target shareholders, an agent (periodically referred to as the "manager" or the "agent"), and a group of potential buyers (all described in greater detail below). Our game has four periods (indexed by $t \in \{0,1,2,3\}$) with no time discounting. At $t = 0$, the policy variables establishing corporate governance and shareholder appraisal actions are determined. Also the auction format, including a possible reserve price, is established by the manager. At $t = 1$, bidders privately observe their respective valuations of the target, and bid on the company in an auction designed by the manager. At $t = 2$, target shareholders may vote as to whether to accept the winning bid. If a sufficient fraction vote in favor, the transaction is consummated and all shareholders (even those who dissent) are forced to relinquish their shares.[36] At $t = 3$, dissenting shareholders who did not previously vote for a consummated merger may be allowed to choose between (a) accepting the terms of the merger or (b) receiving a judicially determined "fair value" through an appraisal proceeding.[37] We will analyze and compare several alternative approaches for determining fair value (including an appraisal remedy based on the merger price).

The target is owned by a single class of fully-distributed (voting) stock, held by a continuum of diffuse shareholders with an aggregate mass we normalize at 1. Each shareholder is assumed to own an infinitesimal fraction $d\gamma$ of the company.[38] We allow shareholders to place differential valuations on the firm as a going concern (which determines the shareholder's willingness to accept a purchase offer). These differential valuations may be due to any number of factors, such as distinct tax positions, differential needs for liquidity, divergent beliefs, and so forth.[39] Specifically, shareholders come in different types, denoted by $\gamma \in [0,1]$, which indexes each shareholder's willingness to accept ("reservation value"). Accordingly, a shareholder of type γ values the entire firm

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36 For the sake of simplicity, we are assuming a single-step merger (such as that done under DGCL §251(c)), but other transactional formats are also feasible. For instance, under the recently created "medium form" merger statute, assuming other requirements are satisfied, so long as the buyer can induce more than 50% of the target shareholders to tender their shares in the first step, the buyer can cash out the rest of the shareholders at the same consideration. See DGCL §251(h). The analysis under that two-step merger regime will be the same.
Also, for simplicity, we are assuming that the merger consideration is in cash, though the analysis easily extends to other types of consideration, including stock.

37 This is consistent with giving the dissenters the right to withdraw from the appraisal proceeding (and take the merger consideration) after the merger has been executed under DGCL §262(e) (giving dissenting shareholders 60 days, after the completion of the merger, to withdraw from the appraisal proceeding and take the merger consideration).

38 The assumption of continuous voters is common in the literature, and we assert it here strictly as a modeling convenience; it is best interpreted as the limiting case of a large but finite number of $2T + 1$ discrete shareholders, where T is arbitrarily large. In fact, when we analyze the shareholders' strategies and the resulting equilibrium, we will refer back to the discrete case. For related treatments in other voting models, see Patty et al. (2009); Batina and Ihori (2005) at 57; and Gelman et al. (2004) at 85.

39 See, e.g., Stulz (1988) (tax basis differences among shareholders generating different reservation values) and Brunnermeier, Simsek, and Xiong (2014) (players holding divergent beliefs that are common knowledge but do not converge).

at γ, and thus her fractional ownership stake at $\gamma \cdot d\gamma$. Shareholder types are distributed according to a commonly-known cumulative distribution function $H(\gamma)\colon [0,1] \to [0,1]$, with associated density function of $h(\gamma) \in (0,\infty)\ \forall \gamma \in [0,1]$. For the sake of simplicity, we assume that $h(\gamma)$ is continuously differentiable. (We will frequently illustrate our results with the special case in which γ has a uniform distribution, where $H(\gamma) = \gamma$ and $h(\gamma) = 1$).

Shareholders' differential willingness to accept (different "reservation values") naturally induces differences of opinion among them about the attractiveness of any given buyout bid. Accordingly, it will help to distinguish between three shareholder types. The *marginal* shareholder is the shareholder type whose valuation is the lowest among all existing shareholders, and is thus the most willing to sell. Under our normalization assumptions, the marginal shareholder is the one with the lowest type of $\gamma = 0$. Note that this marginal shareholder is also functionally the price maker for the market, since its valuation determines the market price (normalized to zero) for shares in the absence of a material prospect of a merger.

Shareholders with types $\gamma \in (0,1]$, by contrast, systematically value the company more than does the marginal shareholder, and they are thus more reluctant to sell. Aggregating across the entire distribution of shareholders, the total valuation that incumbent shareholders place on the company as a going concern (i.e., in the absence of material acquisition prospects) is given by:

$$\int_0^1 \gamma\, h(\gamma) d\gamma = E(\gamma) \equiv \mu \in (0,1)$$

Our assumptions about $h(\cdot)$ guarantee the existence of a shareholder with type $\gamma = \mu$, whose personal assessment of the target's value coincides with aggregate valuation across all incumbent shareholders. We will refer to this shareholder as the *representative* shareholder.

Finally, let $\rho \in (0,1)$ denote the *pivotal shareholder* who provides the swing vote on the desirability of a merger. The pivotal shareholder's valuation is directly determined the critical threshold of shareholder approval needed to consummate the merger, which we will denote by the parameter $\alpha \in [1/2,1]$. By construction, conditional on a price of $b \in [0,1]$, all shareholders with $\gamma \leq b$ will support selling at that price while shareholders with $\gamma > b$ will oppose the sale. Assuming shareholders vote sincerely (which we will explore below), satisfying the requisite approval threshold α requires that the proposed price (b) be sufficiently high that $H(b) \geq \alpha$. Consequently the shareholder with valuation ρ satisfying the condition $\alpha = H(\rho)$ becomes the pivotal shareholder.[40] (If γ is uniformly distributed, for example, we have $\alpha = \rho$.) While our framework allows the

[40] Note that, with the assumptions on h, the relationship between α and ρ is a one-to-one mapping, given by $\rho = H^{-1}(\alpha)$, where $0 = H^{-1}(0)$; $1 = H^{-1}(1)$; and $H^{-1'}(\alpha) > 0$.

approval threshold α to be set at any level, we will periodically highlight the 50% point coinciding with the median shareholder ($\alpha = 1/2$).[41]

Because of shareholder heterogeneity, the marginal, representative, and pivotal shareholders will generally have different types. Indeed, under our distributional assumptions both $\mu > 0$ and $\rho > 0$, and thus the marginal shareholder must always be the lowest type of the three. The ordering of μ and ρ, however, is indeterminate and hinges on both relevant vote threshold (α) and the shape and relative skewness of $h(\cdot)$. In the case where $h(\cdot)$ is symmetric and $\alpha = 1/2$, for example, it follows that $\mu = \rho$, and the representative shareholder is also the pivotal shareholder. (And when γ has a symmetric uniform distribution, we have $\mu = \rho = 1/2$.)

The agent (a.k.a., the "manager") plays a simple but important role at the target firm. She is entrusted with designing the auction (at $t = 0$) that might culminate in a successful acquisition bid. From an institutional perspective, we view the agent as an amalgam of the target's managerial team along with other actors who assist with the auction (such as financial advisers, legal advisers and investment bankers). In the baseline model, we assume that the agent's chief design role is to establish ground rules for bidding—which we model as setting a "reserve price" $r \geq 0$ that establishes the minimum opening bid. The agent-manager's payoff does not necessarily align with that of shareholders, which causes an agency cost problem. Specifically, the agent receives a payoff of $M > 0$ if the firm is sold (regardless of price), and zero otherwise.[42] Viewed ex ante, the manager's payoff[43] as a function of the reserve price is $M \cdot \Pr\{Sale|r\}$. This payoff structure is meant to represent one type of agency cost frequently associated with company auctions, in which the deal team is overly incentivized to close a deal at a low price.[44]

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[41] Typical corporate law defaults tend to focus on the median voter. (See, e.g., DGCL §251(c), "a majority of the outstanding stock of the corporation entitled to vote"; DGCL§251(h) (allowing two-step squeeze out mergers after procuring at least 50% in a first-step tender offer). There are exceptions, however. As noted above, in traditional two-step acquisitions (prior to the enactment of DGCL §251(h)), the effective threshold in the first step was 90% (i.e., $\alpha = H(\rho) = 0.9$). Also, under Delaware's anti-takeover statute (DGCL §203), an "interested" stockholder who acquires 15% or more a target's stock is not allowed to continue with a freeze out for three years unless it either (1) obtain 85% of the outstanding stock at the time it becomes an interested stockholder or (2) secures an approval from 2/3 of the remaining shareholders. We can think of these thresholds as setting $\alpha = H(\rho) = 0.85$ or $\alpha = H(\rho) = x + 2/3(1 - x)$, respectively, where $x \geq 0.15$ denotes the value of the block interested shareholder's initial claim.

[42] We can think of M as coming from any additional incentive that the manager has, on top of an existing pay-for-performance scheme, in selling the company. It can also incorporate any golden parachute payment that the manager may be entitled to.

[43] There may be other auction-related tasks for the agent, such as recruiting bidders to participate. In fact, we analyze an extension below of the case where the agent both sets a reservation price *r* and expends non-pecuniary effort cost to recruit each successive bidder to the table. We discuss this in the extension section.

[44] See, e.g., Smith v. Van Gorkom 488 A.2d 858 (Del. 1985) (alleging that the Trans Union CEO, nearing retirement and looking for an exit opportunity, sold the target too cheaply and with inadequate diligence). Other actors organizing the sale, such as investment banks, typically receive compensation only if a deal closes. This sum is often magnified further in "stapled financing" deals, where the agent also coordinates financing for the buyer (also for a fee upon closing). See, e.g., In Re. Rural Metro Stockholder Lit., 88 A.3d 54, 63 (Del. 2015). Other types of agency cost no doubt affect corporate managers; often, directors and officers are thought to be too *reluctant* to sell the company. In those cases, however, questions of post-

Turning finally to the interested buyers, we suppose that $N \geq 1$ buyers (bidders) have been recruited to bid in the auction. We assume N to be exogenous at this stage, extending the model in a later section to be endogenous. Each bidder $i \in \{1, \dots, N\}$ costlessly observes its private valuation of the target, denoted by v_i. To fix ideas, we study a private values auction,[45] and we accordingly assume that each v_i is independently and identically distributed on $[0,1]$ according to a commonly-known cumulative distribution function $F(v)$, with associated density function of $f(v) \in (0, \infty)\ \forall v \in [0,1]$. As in the case with $h(\gamma)$, we assume that $f(v)$ is continuously differentiable. We also make a standard regularity assumption that $\frac{1-F(v)}{f(v)}$ is monotone non-increasing in v. Buyers are assumed to maximize their expected payoff in the auction. As with shareholders, we will frequently offer examples in which buyer types are uniformly distributed.

For the purposes of analyzing the auction, it will be useful to define several order statistics associated with buyer valuations. Let $v_{(j)}$ denote the j'th order statistic on the N various realizations of v_i's, where we define $v_{(1)}$ as the lowest realization and $v_{(N)}$ as the highest realization. One order statistic that will play a useful role is $v_{(N-1)}$, corresponding to the second highest realization among v_i's. In a generic second-price (Vickrey) or English auction, the winning buyer's bid will be equal to the second highest realization, conditional on the winning buyer having the highest realization. (This condition, however, will not always hold in our model when appraisal introduces the prospect of two-tier pricing.)

II. Equilibrium Analysis

Having laid out the basics of the model, in this section we present the main equilibrium results, for a variety of combinations of dissenters' rights target shareholders may enjoy. Because our game involves a sequential extensive form game with some privately informed players (including both the buyers and the shareholders), Perfect Bayesian Equilibrium (PBE) is an appropriate solution concept. We use this concept throughout in what follows, referring to it simply as an *equilibrium*.[46]

In proceeding, is important to be mindful that appraisal actions are not the only strategic check shareholders have on the auction process in our model. Most notably, shareholders also usually have a collective right to approve or veto the winning bid, since they are required to vote to approve the winning bid that emerges from a company auction. Consequently, in order adequately to understand the effect of appraisal rights,

acquisition appraisal tend to be of little consequence. Our assumption, then, is perhaps the most natural one for the questions we study.

[45] We discuss how our analysis extends to correlated-values auctions later in the paper.

[46] In some ways, Perfect Bayesian Equilibrium is stronger than what we need, since all equilibria discussed below are independent of players' (shareholders') belief structures about the privately informed player types. That said, in some of the voting equilibria below, we also impose a refinement that admits as equilibria only those strategy profiles that are not weakly dominated. See __ infra.

one must also understand the interaction between appraisal rights and approval rights. We therefore proceed to analyze four distinct cases, as pictured by Table 1 below.

	No SH Approval Right	*SH Approval Required*
No SH Appraisal Right	A	B
SH May Seek Appraisal	C	D

Table 1: Combinations of Shareholder Approval and Appraisal Rights

Our equilibrium analysis will work through each combination from the above table progressively: (A) a" benchmark" case where neither shareholder appraisal nor approval are possible; (B) the case where shareholders vote whether to approve a merger, but no appraisal is possible; (C) the case where individual shareholders can seek appraisal but no shareholder vote is allowed; and finally (D) the case where both shareholder approval and shareholder appraisal are available and interact with one another. As it will turn out, the equilibria will generally turn on which combinations of dissenters' rights are in play. (We will be more explicit about this as we proceed.) In working through cases (C) and (D), we will also assess different valuation metrics for determining fair value, comparing the MP rule to other plausible candidates that do not use the merger price as an input.

A. Benchmark Case: Pure Auction with No Shareholder Voting and No Appraisal Remedy

We begin by studying a benchmark case in which shareholders have a simple (if unattractive) role: they have no voice whatsoever, and everything is left to the agent-manager to design, implement and close the sale. This benchmark case is not only simple (because the shareholders' strategy profile becomes irrelevant), but it underscores the potential problem posed by agency costs in executing the sale.

As assumed above, $N \geq 1$ bidders participate in the auction. We assume the auction to be an ascending-bid (English) auction, in which a commonly observed bid opens at a reserve price $r \geq 0$, and continuously rises until the earliest moment where a single bidder remains active, and she immediately wins and pays the prevailing price (stopped bid). If there are no active bidders at the opening price, then no sale occurs. (As is well known, this structure is revenue equivalent to a second-price, Vickrey mechanism in which all bidders submit sealed bids of their willingness to pay—including the seller, who bids the reserve price—and the highest bidder wins the auction paying the second highest bid or the reserve price.)[47]

The equilibrium of the auction game is well known in the literature. (We provide a refresher of key results in the Appendix.) Truth telling by bidders is dominant strategy in equilibrium, and thus each buyer stays in the auction until the bid surpasses his

[47] In fact, in the unaffiliated and private value setting, all four standard auctions, first-bid, second-bid, English, and Dutch, produce the same revenue for the seller. This is known as the revenue equivalence principle. See Milgrom and Webber (1982) and Krishna (2002) at 29—36.

valuation v_i. (See Myerson (1981), Milgrom and Webber (1982), and Ausubel and Cramton (2004)). The probability of a sale for a fixed $N \geq 1$ number of bidders and reservation price $r \geq 0$ is equal to the probability that at least one bidder's valuation exceeds the reserve price:

$$\Pr\{Sale|N,r\} = 1 - F(r)^N$$

Note that this is increasing in N, but decreasing in r. Given N and r, the selling shareholders' corresponding expected aggregate payoff is (see Appendix):

$$Nr[1 - F(r)]F(r)^{N-1} + N(N-1)\int_r^1 [1 - F(y)]F(y)^{N-2}ydF(y) + F(r)^N\mu$$

An important question that will animate much of our discussion concerns the "optimal" reserve price $r^* \in [0,1]$ that maximizes the shareholders' expected aggregate payoff. A well-known (and somewhat surprising) result from the literature is that r^* is independent of the number of bidders, and is characterized by the following relationship:

$$r^* = \mu + \frac{1 - F(r^*)}{f(r^*)}$$

Note that since $\mu \in (0,1)$ and $f(\cdot) < \infty$, $r^* \in (\mu, 1)$. Thus the shareholders optimally set a reserve price that exceeds their aggregate (average) valuation. The condition above is closely related to the monopoly pricing problem, in which the seller determines price by balancing the chance of no sale against the hope of a higher winning bid. If the shareholders could choose their own reserve price, then r^* would be a logical choice (a point we return to in the next section).

Unlike *Priceline.com* customers, however, shareholders cannot name their own reserve price. Rather, it is the *agent* who controls the auction process. Recall that the agent's expected payoff is:

$$M \cdot \Pr\{Sale|N,r\} = M \cdot (1 - F(r)^N)$$

This payoff is strictly decreasing in r, since increasing the reservation price enhances the chance that no bidder is willing to post the opening minimum bid. This observation immediately spawns the following proposition (all proofs are in the Appendix):

Proposition 1. *When neither shareholder approval nor appraisal are available, there is a unique equilibrium in which the agent sets the reserve price at* $r_m = 0 < \mu < r^*$. *An acquisition always occurs in equilibrium, yielding an expected payoff to target shareholders equal to:*

$$E(v_{(N-1)}) = N(N-1)\int_r^1 [1 - F(y)]F(y)^{N-2}dF(y)$$

This expected payoff can be less than shareholders' status quo value of μ, *and it is strictly lower for* N *sufficiently small.*

The intuition behind Proposition 1 is relatively straightforward. If shareholders have no voice in monitoring the sale process, then the agent is free to maximize her own utility by designing an auction that maximizes the probability of a sale. This task is uniquely accomplished with a reserve price of zero, which ensures that a transaction will always occur, albeit at a depressed price relative to what would obtain in an optimally designed auction. Not only is this transaction sub-optimal from the target shareholders' perspective, but they might actually be *worse off* from it, depending on the number of bidders and the underlying distribution of $H(\cdot)$ relative to $F(\cdot)$. (For example, if there is only a single bidder, the shareholders receive expected payoff of $0 < \mu$.[48])

B. Shareholder Approval but No Appraisal Remedy

Now consider the case where shareholders are allowed vote to approve (or veto) a sale to the winning bidder, but appraisal rights remain unavailable. (We assume that failure to procure a sufficiently high fraction $\alpha \equiv H(\rho)$ of affirmative votes ends the game, with shareholders consuming their status quo payoff and all others receiving nothing.) Just as before, the manager begins by designing the auction (through announcing $r \geq 0$), and just as before the $N \geq 1$ participants in the auction will have an incentive to bid truthfully, dropping out only when the (commonly observed, ascending) bid exceeds their individual valuation. Nevertheless, the requirement of shareholder approval adds an important wrinkle. Now, the winning bid must also be acceptable to the pivotal voter, who values at ρ.

As is well known in the political economy literature, voting models with large numbers of players generically have multiple (and possibly infinitely many) equilibria. The usual culprit is indifference: for any posited voting equilibrium where a clear winner emerges, no single player's vote is "pivotal" in determining the outcome. Knowing this, each voter finds herself indifferent about how to cast her vote—so much so that she is willing to vote for outcomes she disfavors. This problem carries over to our setting too, since shareholders are distributed on a continuum. Accordingly, for each winning bid b, our voting game stage has multiple (in fact, infinitely many) Nash equilibria, in which shareholders can either approve or veto a proposed merger, regardless of their preferences and regardless of the price offered. Although a standard refinement will ultimately come in handy in eliminating all but the most intuitive of these equilibria, the multiplicity problem warrants explicit recognition here, as it bears on questions of institutional design. Shareholder voting *may* be a capable institutional elixir to treat managerial agency costs, but this check is only as good as one's faith that the voting

[48] When the manager must expend effort to recruit bidders, the $N = 1$ case may be a real possibility, since the manager can secure a sale after finding only one bidder. We return to this possibility in the extension section, where we generalize the model to allow for N to be endogenously determined through the manager's effort.

equilibria that ensue are "well behaved" and predictable. In certain settings, such faith may be misplaced.

That said, there is a standard refinement in voting theory from binary-outcome voting games that makes intuitive sense here and permits us to narrow the set of equilibria substantially. The refinement involves the *elimination of weakly dominated strategies* (See Duggan (2003)). Informally, a strategy is a weakly dominated strategy if some alternative strategy always makes the player at least as well off regardless of others' actions, and in some imaginable situations that strategy makes the player strictly better off. More formally, this refinement disallows any posited equilibrium strategy $\hat{\sigma}_\gamma$ for any player γ if there exists an alternative strategy $\tilde{\sigma}_\gamma \neq \hat{\sigma}_\gamma$ that fares at least as well for player γ across every possible permutation of opponents' (other players') strategies $\boldsymbol{\sigma}_{-\gamma} \in \boldsymbol{\Sigma}^{-\gamma}$, and does strictly better for player γ in at least one such permutation. We will refer to the set of equilibria that remain after removal of weakly dominated strategies as *weakly undominated equilibria*.[49]

In the case of voting over binary choices, a weakly undominated equilibrium always exists and can be used to eliminate equilibria involving voters casting support to their least preferred choice. In this model, shareholders face a binary choice, but are atomistic and effectively indifferent about how they vote in *almost* all conceivable circumstances. However, in one knife edged situation (where the other shareholders are split evenly between approval and rejection), the shareholder becomes pivotal and has a strict preference to vote for the outcome she prefers.[50] This "sincere" voting strategy is the unique one of a simple voting game in weakly undominated strategies, and it therefore survives the refinement. (As will become apparent below, sincere voting need not carry to the permutation of our model where voting and appraisal interact.)

Under this refinement, the preferences of the pivotal shareholder with valuation $\gamma = \rho$ begins to loom large. In particular, if the highest bid is less than ρ, then the pivot and all those with types $\gamma \geq \rho$ will disfavor it, vote against the merger, and no transaction will be consummated. Only bids that offer at least the pivotal shareholder's value are feasible. Effectively, then, shareholder approval imposes a constraint on the range of "reserve prices" the manager-agent may credibly impose, effectively constraining her to set $r \geq \rho$. This modification stands in stark contrast with the earlier situation, where the agent was unconstrained and set $r = 0$. With the voting constraint, the agent's payoff remains strictly decreasing in r, and thus her optimal strategy is to set r as low as possible, $r_m = \rho$. (In the case of a uniform distribution for shareholders' valuations, this would imply that $r_m = \rho = \alpha$.) Because there is by assumption no appraisal remedy, conditional on there being an approved merger, all shareholders will be cashed out for the same consideration.

[49] Without loss of generality, we impose a convention in what follows that when a player's payoff is the same under a merger and no merger, that player breaks the tie by voting for the merger.
[50] Formally, the shareholders' "pivotality" is established by viewing the continuum of shareholders as a limiting case of finite, discrete shareholders, as noted above.

Recognizing that ρ is now a *de facto* reserve price for the auction, two qualitative outcomes are possible. First, all buyers' valuations fall below ρ, in which case no sale takes place. Second, there is at least one bidder for whom $v_i \geq \rho$, in which case a sale will occur at a price of $min\{\rho, v_{(N-1)}\}$. This logic immediately generates the following Proposition.

Proposition 2. *When a fraction of shareholders of at least* $\alpha \in [1/2,1]$ *is required to approve a merger but appraisal is unavailable, there is a unique weakly undominated equilibrium in which shareholders vote sincerely and approve the merger if and only if the winning bid is at least equal to* ρ. *The agent sets the reserve price at* $r_m = \rho$. *The equilibrium probability of a sale is* $1 - F(\rho)^N < 1$, *and the expected equilibrium payoff to target shareholders is equal to:*

$$N\rho[1 - F(\rho)]F(\rho)^{N-1} + N(N-1)\int_{\rho}^{1}[1 - F(y)]F(y)^{N-2}ydF(y) + F(\rho)^N\mu$$

The equilibrium payoff must exceed the shareholders' payoff in the absence of approval (Proposition 1) if $\rho \leq r^*$, *and exceeds the status quo value of* μ *whenever* $\mu \leq \rho \leq r^*$.

Although the discussion above treats ρ as given (say, because it is enshrined in the applicable jurisdiction's merger statute), that need not always be the case. Some companies, for example, have baked into their charters a "supermajority" provision, which conditions fundamental changes on a larger level of shareholder support. It would be possible, in theory, for a company to adjust its charter to set $\rho = r^* \Leftrightarrow \alpha = H(r^*)$, so that even without appraisal, shareholder approval alone might support an optimal reserve price. Such advance planning may not be possible, however, when the optimal reserve price hinges on the future distribution of buyer valuations, which itself may not be known *ex ante*.

An alternative way to adjust ρ is through the structure of the merger transaction itself. For example, recall that the alternative structures available for executing a two-step squeeze out merger require that either 90-percent or 50-percent of target shareholders (respectively) assent to the first step tender offer. Structuring a deal as one or the other of these approaches effectively commits the vote to meet either a super-majority or bare-majority threshold. Alternatively, the merger agreement itself may condition closing on receiving a supererogatory threshold of votes and/or a maximal threshold of dissenters. Such supermajority provisions are not uncommon in negotiated acquisitions.[51] That said, it would be up to the agent—perhaps working with the group of bidders—to opt into the structures described above; and, also as noted above, both groups have incentives to push the reserve price as low as possible. There is reason to be skeptical that, as a general matter, merger agreements will systematically protect target shareholder value (since the agent benefits by setting the reserve price as low as possible). That said, it is possible that

[51] In fact, a related provision, known as a "blow" provision, has a similar effect, allowing the buyer to back out if more than a critical mass of shareholders seek appraisal. We discuss blow provisions in the extension section.

the prospect of other forms of shareholder litigation (alleging, for instance, *Revlon* claims or breach of fiduciary duty) may imperil the deal unless it is sufficiently "sanitized" by subjecting the proposal to a super-majority vote by the shareholders. (We return to this possibility in the extension section.)

1. Uniform Distribution ($f(\cdot) = h(\cdot) = 1$) Example

It may be helpful to think through the effects of Proposition 2 more concretely with a uniform distribution assumption. Suppose both the target shareholders' and the bidders' reservation values are uniformly distributed: $f(\cdot) = h(\cdot) = 1$. Under simple majority voting requirement ($\alpha = 1/2$), we know that $\rho = \mu = 1/2$. With respect to the bidders, $v_{(N-1)}$ follows a beta distribution with parameters of $N-1$ and 2: $v_{(N-1)} \sim B(N-1,2)$ and $G(y) = \frac{B(y,N-1,2)}{B(N-1,2)}$. The unconditional expected value of $v_{(N-1)}$ is given by:

$$E\left(v_{(N-1)}\right) = \frac{N-1}{N+1}$$

From before, the seller's expected profit is given by:

$$N \cdot E_v\{m(v;r)\} + F(r)^N v_0 = N \cdot \left(\frac{1}{4} \cdot G(1/2) + \frac{1}{2}\int_r^1 y \cdot g(y)dy\right) + \frac{1}{2}^{N+1}$$

In this case, the target corporation with the reservation value of $\mu = 1/2$ would maximize expected shareholder payoff by setting the reserve at $r^* = 3/4$, which is independent of the number of bidders. Another possibility is to set a supermajority approval requirement. If the proposed merger is subject to a 75% supermajority requirement, even with $r = 0$, the target shareholders will maximize their expected return from the auction.

C. Appraisal with No Shareholder Approval

We now switch to the case where an appraisal option is available to shareholders, but there is no shareholder vote to approve. While seemingly unlikely at first blush, this scenario can sometimes occur in practice, such as in the context of acquisitions of controlled firms or squeeze outs.[52] (It is, in any event, an instructive case to consider on a technical level before combining appraisal with shareholder voting.)

Without a shareholder vote, the outcome of the auction fully determines whether a sale occurs, subject to the terms laid out by the agent-manager. Should the highest bid exceed the manager's stated reserve price r, the highest bidder purchases the company at a price of $min\{r, v_{(N-1)}\}$. However, once a winner and a price are determined, all

[52] See, e.g., DGCL §253 ("short-form" merger). The charter can also take away the voting rights to certain class (but not all classes) of shareholders, such as the class of non-voting preferred stock.

shareholders may choose[53] between (a) taking the merger consideration and (b) petitioning the court to determine the "fair value" of their shares, which we denote as $\phi \in [0,1]$. We assume this amount is paid by the successful buyer in lieu of the merger consideration to all petitioning, dissenting shareholders, while the non-petitioning (approving) shareholders are cashed (squeezed) out on the terms of the merger.

A key question in determining the equilibrium in this case is the approach undertaken by the court to assess fair value. Our analysis compares two types of regime: (1) the MP rule, where, ϕ is set equal to the winning bid in the auction, floating up and down with the merger price; and (2) a conventional rule, where, ϕ is fixed ex ante, and its terms are independent of the realized bids in the auction. We analyze each rule, in turn, below.

1. The Merger Price (MP) Rule

First consider the effects of the MP rule, so that the appraisal value is pegged to the merger price such that $\phi = min\{r, v_{(N-1)}\}$, moving mechanically up and down with the winning auction price. Against this backdrop, consider a shareholder of type γ who is choosing between accepting the terms of a merger at a winning bid of $b = min\{r, v_{(N-1)}\}$ or seeking a judicial appraisal and receiving the same amount $\phi = b$. This shareholder gains nothing from seeking appraisal over simply accepting the merger terms. And in fact, from the winning buyer's perspective there is no difference either: the consideration—and the consequences—are identical regardless of the shareholders' strategy.

The strategic role that the MP rule serves in equilibrium is significant: because the MP rule provides no natural "outside option" to the merger price, it is unable to create a constraint on reserve prices, thereby leaving such choices completely up to the agent. The resulting strategic landscape, then, is identical to the benchmark case (from subsection A), where there was neither appraisal nor approval. And, just as in that benchmark case, the agent-manager favors the lowest reserve price she can credibly commit to ($r = 0$), since her payoff is strictly decreasing in r. We therefore arrive at the following result.

Proposition 3. *When appraisal is available at an amount pegged to the merger price, but there is no shareholder approval, the unique equilibrium is identical to the case where neither appraisal nor approval were permitted (Proposition 1). The manager-agent sets the reserve price at* $r_m = 0$*, and a merger always occurs in equilibrium. The parties' expected payoffs are the same as in Proposition 1.*

Proposition 3 embodies an important intuition that merits emphasis. At least when viewed in isolation, the MP rule is tantamount to eliminating the appraisal remedy

[53] Formally, appraisal is available only to shareholders who did not vote in favor of the merger. See DGCL §262(a). That is true by construction here, since no vote occurs. It also holds in various types of short form mergers.

altogether, and with it whatever price protection that it might be able to support under alternative forms of measurement. We proceed to address such alternatives now.

2. The Conventional Rule

Now consider a "Conventional" rule in which the appraisal value is pegged at some fixed value ϕ computed by the court. We will assume that ϕ is set by the court at $t = 0$ and is known by all the players. There is currently significant debate in the literature about *exactly* what this fixed sum should represent, other than the fact that it cannot reflect buyer-specific "synergies."[54] That said, at least some possibilities suggest themselves. First, fair value might be pegged to the valuation attached by the representative shareholder: $\phi = \mu$. Such a measure has intuitive appeal, since it represents the truest measure of the target's going-concern value, aggregating the valuations of all its incumbent shareholders. Another alternative, however, is that ϕ might be pegged to the minimal reserve price that a seller with value μ would demand in an auction with an arbitrary number of bidders. Recall that regardless of *N*, the target corporation maximizes its expected return in the auction by fixing the reserve price at $r^* = \mu + \frac{1-F(r^*)}{f(r^*)}$, below which no sale would occur. The court might navigate this same narrative, presuming that a well-managed corporation would never sell *to any buyer* unless the reserve price were at least equal to r^*. Both of these candidates avoid capitalizing deal synergies from any specific buyer, and one could imagine either as the product of a discounted cash flow analysis (depending on the nature of forward looking projections). Nevertheless, we remain generally non-committal at this stage about which specific vision courts can/do/should pursue, since the analysis is similar for each candidate. Note, however, that all of the above candidates share the trait that (unlike the MP rule) they are extracted from fundamentals of the model, rather than from observable equilibrium behaviors.

Consider a shareholder of type γ who is choosing between accepting the terms of a merger, or seeking a judicial appraisal and receiving price ϕ, computed using the various considerations above. Regardless of the shareholder's type, it is clear that she would seek appraisal in all cases where the merger price falls below ϕ, and will accept the merger terms otherwise. Just as with shareholder voting, the Conventional rule imposes a lower bound on the range of minimally acceptable "reserve prices" the agent may credibly demand, effectively requiring her to choose $r \geq \phi$. All the analysis from Proposition 2 then follows but with ϕ rather than ρ: the manager will optimally set $r = \phi$, and the outcome of the auction will be either no sale, or a sale at a price equal to $min\{\phi, v_{(N-1)}\}$. The following proposition immediately follows.

Proposition 4. *When appraisal is available under the Conventional rule in the amount ϕ but there is no shareholder approval, there is a unique equilibrium prescribing that the manager set the reserve price equal to the appraisal value ($r_m = \phi$). The probability of*

[54] See, e.g., DGCL §262(h) (requiring the Chancery Court to determine the "fair value of the shares exclusive of any element of value arising from the accomplishment or expectation of the merger or consolidation").

a sale is $1 - F(\phi)^N < 1$ *in equilibrium, and the expected equilibrium payoff to target shareholders equal to*

$$N\phi[1 - F(\phi)]F(\phi)^{N-1} + N(N-1)\int_{\phi}^{1}[1 - F(y)]F(y)^{N-2}y dF(y) + F(\phi)^N \mu$$

which is maximized by setting $\phi = r^*$. *Shareholder welfare is strictly greater than the benchmark case (Proposition 1) for all* $\phi \in (0, r^*]$, *strictly greater than the status quo value of* μ *whenever* $\phi \in (\mu, r^*]$, *and strictly greater than under shareholder approval (Proposition 2) when* $\phi \in (\min\{\rho, r^*\}, \max\{\rho, r^*\})$.

Note that the main result is very close to that of Proposition 2, other than the replacement of ρ with ϕ. (In addition, the equilibrium in Proposition 4 does not depend on restricting the strategy space to weakly undominated profiles, since there is no voting stage.) The size of the shareholders' payoff in the appraisal only case depends critically on how ϕ and ρ are set. Note that if the court is free (and capable) to choose ϕ near r^*, then target shareholders likely fare better when limited to appraisal only (Proposition 4) than when limited to approval-only (Proposition 2). On the other hand, if the voting rule induces the pivotal voter ρ to be near r^*, the opposite can hold, and an approval-only regime can dominate.

3. Comparison of MP and Conventional Rules

With the above results in hand, we can offer a preliminary side-by-side comparison of Proposition 3 and Proposition 4, asking how well the MP rule stacks up against plausible Conventional rules within the appraisal-only regime. (We hold in abeyance—until the next section—how adding shareholder voting affects things.) It is easy to see that this doctrinal battle of the bands is not especially flattering for the MP rule. Indeed, as Proposition 3 demonstrates, the MP rule leaves target shareholders in the same position as if they had no dissenters' rights whatsoever. In contrast, the most plausible candidates for Conventional appraisal remedies do strictly better than the MP rule. In fact, even if a court were wildly incompetent, unable to discern with sufficient accuracy any of those plausible measures*,* it could still enhance target shareholder welfare beyond what the MP rule promises simply by fixing fair value at a trivially low level (such as 1¢). Doing so would at least marginally bolster the reserve price beyond zero (the price that the MP rule implicitly advertises).

That said, it is important to recognize that while the MP rule unambiguously depresses share price and target shareholder welfare regardless of N, its disadvantages decline when the number of bidders grows. To see this, see Figure 1, which plots the expected welfare of target shareholders, as a function of the reserve price (horizontal axis) and the number of bidders (depth axis). The figure assumes that the uniform distribution governs both the target shareholders' and the buyers' valuations. Note the hyperplane cutting through the figure at the point where the reserve price is equal to $r = 0.75$, which is the optimal reserve price in the case. The MP rule effectively reduces this reserve price to zero, represented by the far-left wall of the graph. As demonstrated above, the MP rule represents a significant hit to target shareholder payoffs when the number of bidders is somewhat small (e.g., in the low single digits). However, as the number of bidders increases, the penalty visited by the MP rule shrinks substantially. As we move deeper on the Number of Bidders (the depth) axis, the target shareholders' return around the reserve price of $r = 0.75$ gets flatter. While the MP rule is still unambiguously worse than the optimal reserve price, the penalty shrinks to near zero.[55]

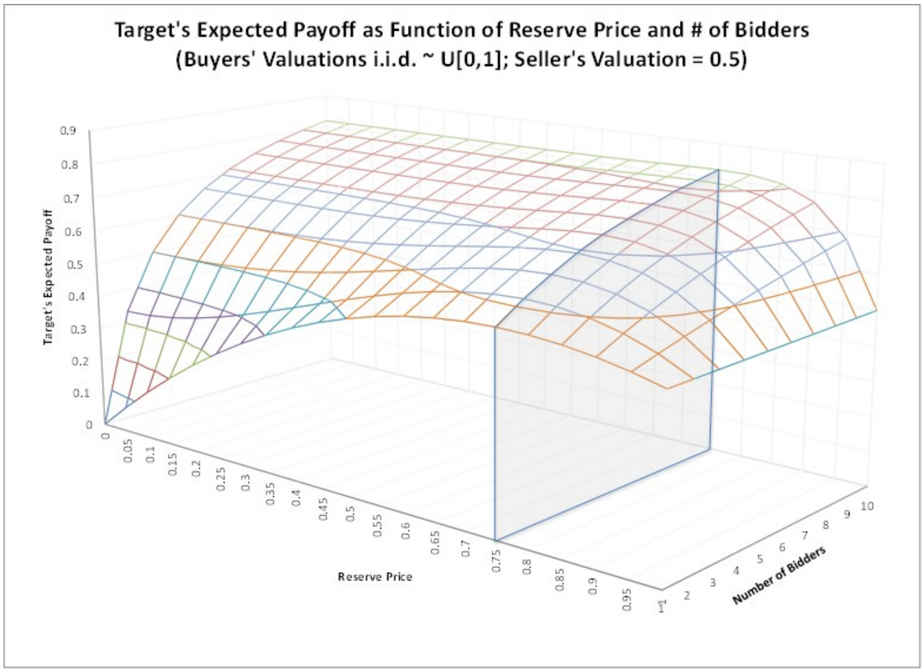

Figure 1: Target Shareholders' Expected Payoff

The intuition behind this example extends the general case too, and it is straightforward to show that the marginal value of increasing r above zero attenuates quickly as N grows, and that this marginal return eventually becomes zero when N is arbitrarily large (as $N \to \infty$).[56] Thus, in the case of appraisal with no voting, the adoption of the MP rule over plausible alternatives—while always harmful to shareholders in a *qualitative* sense—imposes somewhat limited harm on them in a *quantitative* sense when the number of bidders grows large. We revisit this point below.

D. Shareholder Approval Combined with Appraisal

Finally, we consider the hybrid case where both shareholder approval is required and appraisal is available to dissenters. This is (perhaps unsurprisingly) the most complicated case, since we must consider *not only* the effects of both options in isolation, but also their interaction. Such interaction, in fact, is a virtual certainty; for voting and appraisal are designed by law to depend structurally on each other in two related respects.

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[55] Bulow and Klemperer (1996) demonstrates this effect more generally, showing that the value of a reserve price can be swamped by the value of adding another bidder.
[56] For a formal proof of this claim, see Lemma A.1 in the Appendix.

First, under most states' laws (including Delaware's), target shareholders are ineligible to seek appraisal if they previously voted in favor of the merger.[57] In the context of our model (which assumes away abstention), this implies that shareholders must vote against the merger to be eligible. Second, shareholders who vote against the merger receive a true option—they may select whether to take the merger consideration or seek appraisal. Thus, voting in favor of a merger extinguishes real option for the target shareholder, while voting against does not.[58]

Accordingly, now each shareholder's strategy (σ) consists of two elements: (1) determining whether to vote in favor of or against the merger; and (2) in cases where the shareholder has voted against a consummated merger, determining whether to accept its terms or seek appraisal. Unlike the case of pure approval, the combination of these factors can support equilibria with strategic (insincere) voting by shareholders who support the merger yet nonetheless demur, hoping to preserve eligibility to seek appraisal. To analyze this interaction, we once again proceed sequentially, starting with the "Merger Price" rule, and then moving on to several plausible "Conventional" rules. Throughout the analysis, we continue to make the simplifying assumption that a successful merger requires a bare 50% majority vote.

1. The Merger Price (MP) Rule

As in the previous subsection, MP rule is straightforward to analyze. Just as before, the appraised fair value floats mechanically up and down along the with the merger price, so that $\phi = min\{r, v_{(N-1)}\}$. And, just as before, no shareholder ever gains from seeking appraisal, implying that appraisal rights do not affect how any shareholder votes. In this case, the game devolves into the pure approval rights case, similar to that studied above. This reasoning immediately implies the following:

Proposition 5. *When shareholders must approve the winning bid and appraisal is available under the MP rule, there is a unique weakly undominated equilibrium identical to that stated Proposition 2, in which the manager sets the reserve price at the pivotal voter's valuation ($r_m = \rho$). The probability of a sale and the expected equilibrium payoff to target shareholders is identical with Proposition 2.*

The result is not surprising. In the pure approval regime studied above, an underlying assumption was that dissenting shareholders are to be frozen out at the same consideration as the assenting shareholders. This is functionally identical to having a court grant "fair value" appraisal equal to the merger price (the winning bid). Furthermore, given that the buyer only needs to induce the pivotal voter to vote in favor of the bid, the minimum bid is once again equal to ρ. Note that when $\rho < \mu$, it is possible for a buyer to acquire the company for less than its incumbent shareholders

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[57] See, e.g., DGCL §262(a) (appraisal available to shareholder who "neither voted in favor of the merger or consolidation nor consented thereto in writing").
[58] See, e.g., DGCL §262(e) (allowing dissenting shareholder, who previously notified the corporation its intent to exercise the appraisal remedy, to withdraw and accept the merger consideration within 60 days of the completion of the merger).

value it. In any event, it remains the case that even in the presence of shareholder approval rights, the MP rule functions largely to negate the effect of the appraisal remedy, leaving approval as the sole source of reserve price protection for target shareholders.

2. The Conventional Approach

Now consider a variety of Conventional valuation approaches, in which the court fixes fair value at some commonly-known level ϕ, which is untethered to the realized merger price. (We will not rehash here the foregoing discussion about how various measures of ϕ might be justified, but we return to this discussion later.) This turns out to be the most complicated case, and much of the strategic interaction hinges on the relative size of the fair value (ϕ) relative to the pivotal voter's value of the firm (ρ). We therefore analyze the two possible orderings in sequence.

i. Case A: $\phi < \rho$

Suppose first that the appraised fair value is strictly less than the pivotal shareholder's assessment. In this case, appraisal is unattractive relative to the status quo for the pivotal shareholder, since it would pay her less than her going concern valuation. This shareholder, moreover, would oppose (in a "weakly dominant strategy" sense) any bid that offered her between ϕ and ρ. For similar reasons, she would also oppose any winning bid that promises her less than what she would get in appraisal. This leaves only the case where the winning bid exceeded the pivotal shareholder's valuation (so that $b \geq \rho$), which she (and all others with lower valuations) accept. Ultimately, then, in this region the appraisal remedy is insufficiently lucrative to affect the pivotal voter's behavior, as reflected in the following lemma.

Lemma 6A. *Suppose shareholder approval and appraisal are available, and that the Conventional appraisal remedy is such that $\phi < \rho$. All weakly undominated equilibria are identical to that in Proposition 2, where the merger succeeds if only if $b \geq \rho$ where b is the winning bid. The manager therefore sets the reserve price at ρ. The probability of a merger and the expected equilibrium payoff to target shareholders is identical under both regimes.*

ii. Case B: $\phi \geq \rho$

Now consider the somewhat more interesting (but trickier) case where the fair value equals or exceeds the pivotal shareholder's type. In this case, a successful merger can result in an appraised fair value that the pivotal shareholder would find attractive relative to the status quo. Whether (and how) the shareholder responds to this incentive, in turn, depends on the value of the winning bid/merger proposal (which we denote as b).

On one end of the spectrum, suppose b were even higher than the appraisal value (i.e., $b \geq \phi$); in this case, no shareholder would ever seek appraisal since the terms of the merger dominate, and a strong majority of shareholders support (and would vote for) the merger. Here, then, the appraisal option does no added work, and the merger is supported by a strong majority, all voting sincerely. The merger always succeeds.

At the other end of the spectrum, suppose the bid is relatively low (i.e., $b < \rho \leq \phi$). Here, the pivotal shareholder has divided interests: appraisal looks extremely attractive, while the merger price is unattractive (since b is below her status quo payoff). The pivotal shareholder's most preferred option would be to see merger consummated over her "no" vote and then to seek appraisal, and if that route were unavailing, she would want the merger to fail. Either way, her optimal strategy is clear: she finds it weakly dominant to cast her vote against the merger. Similar reasoning also applies to all shareholders who have a higher valuation than the pivot. Consequently, the merger always fails. Summing up so far, when the winning bid is on the low or high end of the scale relative to ϕ and ρ, the equilibrium outcome is relatively straightforward.

Lemma 6B.1. *When $\rho \leq \phi \leq b$, the unique weakly undominated equilibrium prescribes sincere voting and approval of the merger. When $b < \rho \leq \phi$, the unique weakly undominated equilibrium prescribes sincere voting and rejection of the merger.*

The hardest case is when the winning bid resides in the Goldiloxian middle ground where $\rho \leq b \leq \phi$. Here, the pivotal shareholder is attracted to the winning bid, but she is *even more* enchanted by the high appraisal amount. Thus, the pivotal shareholder would most prefer that the transaction be consummated, but then to seek appraisal. However, her next most preferred strategy would be for the merger to be approved and to receive the winning bid *b*. Her least preferred strategy is the outright rejection of the merger. And herein lies the rub, for the pivotal shareholder has countervailing motives: in order to retain eligibility for her *most* preferred outcome (appraisal), she must vote insincerely for her *least* preferred outcome of (rejection). Such an appraisal-preserving negative vote would be acceptable to the pivotal shareholder *if* she could count on other shareholders to carry the requisite majority ($\alpha \equiv H(\rho)$). But alas, all shareholders with valuations on the interval $[0, b]$ are performing the same strategic calculus, hoping that others will vote to support the merger so that they will be able to seek appraisal. A collective action problem thereby ensues, and much depends on whether merger-supporting shareholders coordinate on a voting equilibrium that determines who can seek appraisal, and who must "take one for the team" to approve the merger.

Consequently, two distinct classes of equilibria emerge in this case, which turn on whether shareholders can solve their coordination problem. They are summarized in the following Lemma.

Lemma 6B.2. *When $\rho \leq b < \phi$, there are two classes of weakly undominated equilibria:*

(A) In the first, a coalition of shareholder types $Z_2 \subset [0, b]$ comprising <u>strictly less than an α-fraction</u> of shareholders vote to approve the merger, and all others vote against. All voting against seek appraisal. The merger never succeeds.

(B) In the second, a coalition of shareholder types $Z_1 \subseteq [0, b]$ comprising <u>*exactly an α-fraction*</u> *of shareholders vote to approve the merger, and all others vote against. All those voting against seek appraisal. The merger always succeeds.*

As Lemma 6B.2 illustrates, the eligibility rules for appraisal interact non-trivially with shareholders' voting incentives. When the merger price is attractive to the pivotal voter but the anticipated appraisal value is even more tempting, equilibrium turns on whether shareholders on the interval $\gamma \in [0, b]$ who are in the same boat can cobble together a coalition of "yes" votes to support the merger. If they cannot coordinate (as in the first equilibrium), the merger proposal is rejected, even though a majority of all shareholders would have preferred it. When they succeed in coordinating, in contrast (as in the second equilibrium), the merger wins by a hair's breadth, and the "no" voters (jubilantly) seek appraisal at a higher expected price.

Note that in the second, "coordinated" equilibrium, the prospective buyers' bidding strategy must adapt as well. No longer is it optimal to bid truthfully (as is usually the case in Vickrey or English auctions). Rather, the winning buyer must anticipate the possibility of having to pay two different prices: the bid amount to the "have-nots" (the $\alpha = H(\rho)$ fraction voting in favor and receiving *b*), and a premium price to the "haves" (the $(1 - \alpha)$ fraction voting against and receiving appraisal of $\phi > b$). The buyer's total outlay therefore may exceed her bid of *b,* and will instead be equal to $\max\{b, \alpha b + (1 - \alpha)\phi\}$. Consequently, buyers must prepare to adjust their bidding behavior to account for the anticipated "tax" they pay the dissenters for winning bids in the interval between ρ and ϕ. (Indeed, truthful revelation is no longer incentive compatible for buyers in this intermediate range.) Analysis of the foregoing lemmas yields the following central result:

Proposition 6. *When shareholders vote on the winning bid and Conventional appraisal is available, equilibrium turns on the relative sizes of ρ and ϕ:*

(A) When $\phi < \rho$, the unique weakly undominated equilibrium is identical to that in Propositions 2 and 5. The agent sets the reserve price at the pivotal shareholder's valuation ($r_m = \rho$), and all bidders with valuation $\gamma \geq \rho$ participate. Winning bids are approved and no dissenters seek appraisal.

(B) When $\phi \geq \rho$, there are two classes of weakly undominated equilibria:

(1) In the first, the equilibrium is identical to that in Propositions 4, in which the agent sets the reserve price at the appraisal value ($r_m = \phi$), and all bidders with valuation $\gamma \geq \phi$ participate. Winning bids are approved and no dissenters seek appraisal.

(2) In the second, the agent sets the reserve price at the pivotal shareholder's valuation ($r_m = \rho$), but only bidders with valuations $\gamma \geq \alpha\rho + (1 - \alpha)\phi$ participate. Winning bids of at least ϕ are approved and no dissenters seek appraisal; while winning bids on $[\rho, \phi)$ are approved by exactly an α-fraction of the shareholders and all dissenters seek appraisal.

Although Proposition 6 is somewhat long, its content is relatively straightforward and can be summarized in a few intuitions. First, the appraisal rule "matters" only if it is not overshadowed by the natural check of shareholder approval. When $\phi < \rho$, the appraisal option is insufficiently lucrative to move the needle, since the required vote on the merger already ensures a minimum reserve price of ρ. In this context, then, there is no difference between the MP Rule and a Conventional rule, since both are overshadowed by shareholder approval.

However, once the fair value rises past the pivotal valuation ($\phi \geq \rho$), equilibrium behavior changes significantly—not only in shareholders' decisions to seek appraisal, but also in in voting behavior and in auction dynamics. The net effect is to push the reserve price above ρ. How far above turns on which class of equilibrium emerges. In the "non-coordinated" equilibrium (B)(1), shareholders' collective action problem causes them to reject any bid below ϕ, which then becomes the *de facto* reserve price for the auction. When the "coordinated" equilibrium (B)(2) obtains, voting and appraisal interact. This makes sense, because those seeking appraisal must depend on the affirmative voters to carry the majority that makes appraisal possible. Consequently, the minimum price that the "yes" voters can extract in a successful merger (ρ) combines with the minimum price that the dissenters can procure (ϕ). The end product is a mélange of the two, effectively replicating the payoffs of an ascending auction with *de facto* reserve price equal to the α-weighted combination of the two (or $r_\alpha = \alpha\rho + (1-\alpha)\phi$).

Several aspects of the equilibrium described above warrant attention. First, and perhaps surprisingly, the total outlay of the winning bid in the "coordinated" equilibrium (B)(1) is strictly less than its counterpart in the "non-coordinated" equilibrium (B)(2). (That is, $r_\alpha < \phi$). In effect, the shareholders' inability to coordinate ultimately *helps* them, by allowing them to commit credibly to reject any winning bid between ρ and ϕ; and thus, any winning bid (if one emerges) would have to offer no less than ϕ. In fact, so long as $\phi \leq r^*$ (or at least does not exceed r^* by too much, a lack of shareholder coordination also improves expected shareholder welfare.

Second our model predicts appraisal will be far from ubiquitous. When it occurs in equilibrium, it will be systematically pursued only in those circumstances where (a) the anticipated appraisal award exceeds the pivotal voter's type $\phi > \rho$, (b) the "uncoordinated" equilibrium obtains, *and* (c) the winning bid is somewhere between ρ and ϕ. In no other cases are appraisal proceedings an equilibrium phenomenon in our model. Moreover, when appraisal proceedings are observed in equilibrium, our model predicts that *fair value awards should systematically exceed the merger price.* This ordering holds regardless of whether ϕ is set "too high" or "too low" as measured against some relevant value maximization benchmark. It is simply a byproduct of equilibrium selection bias: strategic litigants will not pursue appraisal (and may not even approve a merger) if the appraisal option seems unattractive. Consequently, we have misgivings about arguments put forth by some commentators that the appraisal system may be "broken" because appraisal awards systematically exceed the merger price.[59] Such

[59] See, e.g., Bainbridge (2012) and Hamermesh and Wachter (2005).

evidence may well be probative about whether parties are acting rationally, but it is not a telltale sign of dysfunction warranting the wider application of the MP rule.[60]

III. Applications and Extensions

Having fully characterized the equilibrium of the combined approval and appraisal game, we are now in a position to discuss applications of and extensions to the model. We begin by considering the design of an "optimal" appraisal doctrine within our model, followed by a brief discussion of several other possible extensions.

A. The Optimal Appraisal Rule

Our first application relates to a core motivation behind writing this paper: assessing the conditions under which the MP rule (or something else) would be optimal for a court to utilize. For purposes of this inquiry, we must define what it means to be "optimal," and here we assert it to be the policy that maximizes the expected payoff of target shareholders, holding constant the other parameters of the model. In other words, where would a court optimally set ϕ to maximize expected target-shareholder value in equilibrium?

We must also identify an appropriate domain from which to choose ϕ. It is reasonable to believe that Delaware judges have significant equitable discretion to select ϕ at over a large range of values. Under current law, the Chancery Court need not follow any prescribed procedure other than the general rules of court. But, the statute requires the court to set the fair value amount "exclusive of any element of value arising from the accomplishment or expectation of the merger."[61] Most commentators interpret this limitation as excluding the award of deal "synergies"—components of value that are specific to an individual buyer's firm-specific traits rather than the seller's overall marketability.[62] At the same time, determining how much of the winning price was due to buyer-specific synergies versus overall seller marketability is no mean feat. Indeed, even most recent cases *using the MP rule* have declined to apply a synergy discount due to a lack of clear evidence. In our view, then, it seems reasonable to expect that courts in our model would have sufficient freedom to set ϕ virtually anywhere on $[0,1]$ spectrum they wished (including the merger price). In that case, the following proposition emerges.

Proposition 7. *Suppose the court is free to choose any* $\phi \in [0,1]$*, and that the approval threshold is set at* $\alpha \in [1/2,1]$ *such that* $\rho = H^{-1}(\alpha)$*. The optimal appraisal rule* ϕ^* *is as follows:*

(A) If $\rho > r^*$*, then* ϕ^* *includes both the merger price and any fixed* $\phi \leq \rho$*;*

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[60] Accord Bomba et al. (2014) (asserting similar conclusions from several practitioners' standpoints).
[61] DGCL §262(h).
[62] See, e.g., Merion Capital v. BMC Software, C.A. No. 8900-VCG, at 46 (Del. Ch. October 21, 2015) (likening the exclusion of synergies to the value of the deal that accrues to the buyer with a patent on the bow when it purchases an arrow company).

(B) If $\rho \leq r^*$, then:

(1) $\phi^* = r^*$ if the "uncoordinated" equilibrium in Proposition 6(B)(1) obtains; and

(2) $\phi^* = \min\{1, (r^* - \alpha\rho)/(1-\alpha)\} > r^*$ if the "coordinated" equilibrium in Proposition 6(B)(2) obtains.

Proposition 7 provides a central result from this paper. It states, among other things, that the MP rule *may* be optimal in some circumstances, but only when the pivotal shareholder's valuation, $\rho = H^{-1}(\alpha)$, exceeds the optimal reserve price in the auction, r^*, which recall is characterized by:

$$r^* = \frac{1 - F(r^*)}{f(r^*)} + \mu$$

The condition that $\rho > r^*$ can be demanding, particularly when the voting rule hinges on the median voter (so that $\alpha = 0.5 \Leftrightarrow \rho = H^{-1}(0.5)$), and it requires significant rightward skewness in the distribution of incumbent shareholders' valuations relative to those of bidders. For example, consider the case where buyers are distributed uniformly on the unit interval, so that $r^* = \frac{1+\mu}{2}$. Thus, for $\rho > r^*$to hold, the median shareholder's valuation would have to be more than midway between the representative shareholder value (of μ) and the maximal shareholder value (of 1). While there exist distributions that satisfy this condition, most conventional ones do not. (For example, if target shareholders were also uniformly distributed, then this condition would fail miserably, since $\rho = \frac{1}{2} = \mu < r^* = \frac{3}{4}$.) Moreover, even in those cases where the median shareholder is so heavily skewed upwards as to satisfy this condition, it would imply that most target shareholders have extremely high valuations relative to possible buyers, calling into question whether conditions are ripe for a profitable merger to begin with.

Nevertheless, if a supermajority vote is necessary to approve the merger (say because target corporation's charter requires it, or merger agreement itself calls for it), it would also be possible for the pivotal shareholder's valuation to exceed the optimal reserve price: $\rho = H^{-1}(\alpha) > r^*$. To take one plausible example, recall that two-step mergers in Delaware traditionally required at least 90% of the target shareholders to tender their shares into a first step tender offer before the squeeze-out step was permitted.[63] This is tantamount to setting $\rho = H^{-1}(0.9)$ under a traditional two-step deal structure. Proposition 7 tells us that when the merger requires supermajority voting requirement of this magnitude, either by compulsion or by the pursuit of a certain deal structure, the pivotal voter may exceed the optimal reserve price, and it would be optimal to relax appraisal standards, possibly by adopting the MP rule rather than the Conventional rule.

[63] See DGCL §253. As noted above, the Delaware legislature has significantly changed the landscape on the two-step mergers by enacting DGCL §251(h), which allows a buyer alternatively to merge with the target corporation without target shareholders' vote if more than 50% of the target shareholders tender their shares at the end of the first step tender process (unless a larger percentage is necessary due to supermajority voting requirement). The new "medium form" merger de facto lowers the approval threshold from 90% to 50%. See Choi and Talley (2016) for more details.

When $\rho \leq r^*$, on the other hand, the MP rule is no longer optimal, and instead the optimal appraisal value is a fixed value, either at r^* (in the case of the non-coordinated equilibrium from Proposition 6) or at $min\{1, (r^* - \alpha\rho)/(1-\alpha)\}\}$ (in the case of the coordinated equilibrium from Proposition 6). The former case is intuitive, since it simply fixes the appraisal value at the optimal reserve price for the representative seller. The latter case is more interesting and provocative, since it suggests that the optimal appraisal value should be set *even higher than the optimal reservation price* r^*. The reason for this added premium stems from the fact that in the coordinated equilibrium, the buyer pays two distinct prices to voting supporters and appraisal petitioners, and the notional value for ϕ^* must be calibrated so that the winning buyer's expected outlay is equal to r^* (or a value that is as close to r^* as possible). This means that in the case of the coordinated equilibrium, the value of ϕ^* can grow quite large. Returning to the case where buyers and shareholders are uniformly distributed and the voting threshold is 50%, it is optimal for the court to fix $\phi^* = r^* = 3/4$ in the uncoordinated case, but in the coordinated equilibrium it should set $\phi^* = 1$, pegging the appraisal value at the highest valuation among the shareholders. (In equilibrium, the winning buyer will pay a "sticker price" of 0.5 to a bare majority of shareholders voting yes, and an appraisal value of 1.0 to the remainder who seek appraisal, averaging out to a total outlay of 0.75.)

It is important to acknowledge that this brief discussion has presumed an "optimal" appraisal rule to be one that maximizes target shareholders' *ex ante* expected payoff. While this assumption is no doubt natural (and indeed ubiquitous) within the context of corporate and M&A law, some judges could harbor objectives that coincide with a broader measure of social welfare. If we also adapt our analysis to embrace such capacious desiderata, it would have implications for several of our results, as well as whether courts should grant shareholders the freedom to set reserve price in the first place. Indeed, as discussed above, the optimal reserve price in an English auction is closely related to a monopolist's profit maximizing pricing condition, who deliberately weighs the chance of failing to make an efficient sale on the margin against the reward of a higher price on the infra-margin. Were we to incorporate both buyers' and sellers' expected welfare in the definition of optimality, it would follow that the optimal reserve price sets $r^* = \mu$, so that the company always ends up in the hands of the highest valuing owner (be that the incumbent shareholders or the highest bidding buyer).[64] The key steps of our earlier analysis would go forward for this case in more or less the same way, but with the significant caveat that now we would have $r^* = \mu$. That alteration, in turn, bears significantly on the conditions under which the MP rule may be optimal, since it would be much more plausible to see $\rho \geq r^* = \mu$ (such as in any case where $h(\cdot)$ is symmetric). And in such a case, Proposition 7 implies that the MP rule would be optimal (though not uniquely so).

[64] Somewhat intriguingly, if we were to include the welfare of *the agent* as well, it is conceivable that the optimal reserve price would be even lower than μ, since the agent enjoys a discontinuous welfare benefit upon sale at any price, even though the firm fails to go to the highest bidder.

B. Other Extensions

There are several other extensions and applications of the basic analysis that are worth serious consideration, and we list several of them here (along with conjectures about how they would affect our analysis). The first concerns equilibrium selection. Proposition 6 (including the Lemmas leading up the Proposition) is silent on the question of equilibrium selection. Yet this question is clearly important, in particular for the buyers with valuations between $\alpha\rho + (1-\alpha)\phi$ and ϕ, who may need to decide whether to participate in the auction or stay away. One possible way of confronting this issue is to identify situations where shareholders are likely to be able to solve their coordination problem over voting and seeking appraisal. Here, a plausible predictor is ownership concentration in the firm. Though we have so far assumed that the target stock is completely dispersed, this may be overly simplistic. For many publicly traded companies, even those with multitudes of beneficial owners of stock, there are a relatively modest number of institutional shareholders holding relatively large fractions of the outstanding stock on behalf of these owners. In that setting, the bare majority equilibrium (as laid out in Lemma 6B.2(B)) may be much easier support, compared to a setting where the shares are completely dispersed and the shareholders are wholly unorganized. (Recall, however, that coordination is generally not in shareholders' collective interest in most cases, since the uncoordinated equilibrium generally results in higher prices.)

Other extensions related to equilibrium selection that are also worth exploring. The multiple equilibria described in Proposition 7, for example, may pose challenges when the judge cannot predict ex ante which equilibrium will materialize. To deal with this uncertainty, at least in theory, the appraisal remedy might be contingent on the realized equilibrium: i.e., the judge may be able to "learn" which equilibrium is in play by observing the deal structure and/or voting outcomes before deciding on the appraisal rule. For instance, when the merger agreement sets the threshold (ρ) relatively high, there may be little need for an appraisal remedy that pushes that de facto reserve price even higher (i.e., $\phi > \rho$). In such cases, it might be better for the court to adopt the Market Price (MP) rule, or put some evidentiary weight on the merger price as corroborative of "fair value," so as to eliminate possible distortion that could be caused by the Conventional approach. This intuition suggests that judges might the MP rule when the appraisal follows a two-step merger involving a 90% squeeze out condition than a 50% condition. Or, alternatively, if the vote was a close call, the judge could infer that it is the coordination equilibrium and set the "fair value" under the appraisal at $\phi^* = \min\{1, (r^* - \alpha\rho)/(1-\alpha)\}$. On the other hand, if the merger is approved overwhelmingly, the judge could infer that it was the uncoordinated equilibrium, and sets the appraisal value equal to $\phi^* = r^*$. At attractive feature about such a contingent appraisal system is that it awards more compensation to the dissenting shareholders when the merger seems more "controversial."

We might also extend the analysis by introducing judges as strategic players in the game. Doing so might provide insights about the creeping judicial embrace of the MP rule as a type of collective action problem. Recall from the introduction that at least some of the judicial preference for the MP rule was predicated on the difficultly and inconvenience courts face in divining independent financial valuations from whole cloth. In the eyes of an individual judge, the end product may not be worth the effort. Indeed, assuming buyers/agents had largely expected courts to utilize an optimal conventional appraisal rule, the deals they produce are likely to look *especially* protective of shareholder interests, dampening a judge's incentive to invest in an arduous fairness calculation when market price is clearly reliable. When that judge rationally defects from the conventional rule, however, it visits an externality on others, since future buyers/agents will design their auction against a prospect of an incrementally diluted appraisal rule—an actuarial hybrid of the MP rule and the conventional rule. Although each defection is rational for the individual judge, the collective effect can be to cause excessive judicial use of the MP rule.

Our analysis may also be amenable to studying a variety of different contractual mechanisms that respond to appraisal risk. From the buyer's perspective, appraisal introduces a type of transactional "tax." Not surprisingly, some buyers try to reduce or eliminate such surprises through a variety of contractual terms. One often-observed contractual clause—known as a "blow" provision—allows the buyer to walk away from the deal if a sufficient fraction of target shareholders exercise the appraisal remedy. Especially if the bare majority equilibrium is anticipated (as in Proposition 6), the buyer may have a strong incentive to adopt a "blow" provision so as to protect itself against a cascade of dissenters (who may be demanding cash compensation substantially higher than the merger consideration). A blow provision that is set at, say 20%, will allow the winning bidder to avoid such an outcome. At the same time, blow provisions also implicitly condition the deal on a supermajority vote to consummate the deal (80% in this case, assuming no abstentions). This side effect may ultimately benefit target shareholders, since it requires the buyer to increase its bid to be attractive to a larger majority of shareholders. Other contractual mechanisms include "drag-along" provisions (which require shareholders to vote in favor of the merger under certain conditions and lose appraisal), and "naked no vote" fees, which require the target to pay the buyer a termination fee in the event of a negative vote by shareholders. Viewed in terms of our model, these latter two provisions tend to coordinate shareholders by making no votes costly or ineffectual. All else held constant, they would tend to dilute bids and shareholder welfare.

Still another line of extension concerns the auction environment and the attributes of buyers. Our analysis has focused on the tractable setting of private, independent valuations among the bidders. In many merger settings, however this assumption may be too restrictive, and we intend to extend our analysis allow for systematic correlation among bidder valuations. The common or affiliated private values setting significantly affect our analysis in several ways. Most notably, the optimal reserve price in our setting is relatively aggressive, since buyers' valuations are assumed uncorrelated. Once we relax that assumption, the optimal reserve price will tend to decline.[65] And, if the optimal

[65] If the seller does not have any information that could affect the buyer's valuations, when valuations (or signals) are affiliated, it may no longer be optimal to set a positive reserve price. See Krishna (2002) at 121—124 (failure of the "exclusion principle").

reserve price falls sufficiently low (below ρ), shareholder approval will tend to be the binding constraint and the MP rule might be an efficient response (perhaps one of many), since the MP rule effectively negates the reserve price created by appraisal. Also, as the optimal reserve price declines (falls below μ, for instance), the agency problem of dealing with a manager who is too eager to sell the company will be mitigated.

Furthermore, while we have assumed that the bidders are aware of all the valuation attributes of the target shareholders (ρ and μ), this may not be true in certain settings. In particular, if the target shareholders' valuations might also be affiliated with buyer valuations, several possibilities ensue. The reserve price might be a signal of shareholders' valuation—which may push it back up; and, the outcome of the auction may change shareholders' self-assessments of whether to approve the transaction.[66] While affiliated value auctions are far more complicated to study, such a setting may be the most appropriate when (for example) the auction draws financial (rather than strategic) buyers. It is commonly asserted that financial buyers, such as private equity investors, are focused on improving the firm's performance through broadly value-enhancing moves, such as streamlining operations, adopting more profitable tax structures, and the like rather than exploiting some idiosyncratic, firm-specific synergy. Given that many of their post-merger strategies exhibit commonality, it may be more appropriate to assume common values setting when there are more financial buyers than strategic buyers.

Finally, we have assumed throughout that the number of bidders (N) is exogenous and fixed *a priori*. Here too, we might enrich the model in realistic ways by allowing the agent-manger also to recruit new bidders into the auction (albeit at some non-pecuniary cost of effort). Such an extension would not only add richness into the auction model but also into the principal-agent setup. For example, suppose it costs the agent an effort cost of k to attract each new bidder into the auction. In that setting, an optimal appraisal rule may depend on the number of bidders and in an interesting way: $\phi^* = \phi(N)$ where $\phi(N)$ declines as N increases. Under this structure, the agent would have an incentive to heat up the bidding through recruiting buyers, and the court would reward her efforts by progressively reducing the effective reserve price. One version of this mechanism would be for the court to revert to the MP rule once the agent-manager has recruited a sufficient number of buyers. Such a contingent rule could function as an incentive device for the deal team.

Conclusion

This paper has developed an analytic framework (combining auction design, agency costs, and shareholder voting) to shed light on how best to measure the "fair value" of a target company's shares in a post-merger appraisal proceeding. Appraisal is

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[66] See Cai, Riley, and Ye (2007). The reserve price signaling hypothesis, however, assumes that the seller cannot credibly convey the information it has to the buyers. Generally, in common value settings, where the buyers update their valuation beliefs based on others' information, it is in the interest of the seller to reveal its information to the buyers. See Milgrom and Webber (1982) and Krishna (2002) at 106—108.

an important mechanism not only in protecting the dissenting minority shareholders' rights but also in affecting the *ex ante* merger negotiation and shareholder approval process by imposing a price floor for the possible benefit of all target shareholders. The topic has also received heightened attention recently due to the rise of so-called "appraisal arbitrage" litigation that some institutional investors have strategically deployed. Due partly to concerns about speculative petition activity as well as the mounting challenge generating independent valuations, the Delaware Chancery Court has become more amenable of late of using the merger price itself as an important piece of evidence (or even the sole evidence) in determining "fair value" when certain conditions are met. An evaluation of the MP rule on *ex ante* behavior has been the principal focus of the paper.

Our analysis has several novel implications. Foremost, as a general matter, the MP rule tends to depress both acquisition prices and target shareholders' expected payoff compared to both the optimal appraisal rule and the most plausible interpretations of the conventional approach that determine the "fair value" independent of the merger price. At the same time, our analysis has also suggested specific conditions under which the MP rule may be (at least weakly) optimal, such as when the deal is structurally dependent on super-majority shareholder approval (such as traditional short-term squeeze outs), and as an incentive device to encourage a deal team to recruit a healthy number of interested buyers. These situations square reasonably well with what appear (to us) to be the several contours of the MP rule as it is developed thus far by courts. That said, it remains the case that a special situation would be needed to justify the MP rule over any number of possible conventional approaches. Consequently, if it is to be used at all, the MP rule should be deployed with some caution.

Finally, our analysis sheds light on a host of other interesting debates that surround post-acquisition appraisal. It helps explain, for example, why a healthy majority of litigated appraisal cases using conventional fair value measures result in valuation assessments exceeding the deal price, an equilibrium phenomenon predicted by our analysis and a simple artifact of rational, strategic behavior (not necessarily an institutional deficiency, as some have suggested). In addition, our analysis facilitates a better understanding of the strategic and efficiency implications of recent reforms allowing "medium-form" mergers, as well as various appraisal-related practices, such as blow provisions, drag-alongs, and "naked no-vote" fees.

Appendix A: A Brief Review of Second Price Auctions with Private Values and Reserve Prices

Because some of the analysis in the text draws on auction theory, we review some of the basics here. For more detail, see the classic treatments by Myerson (1981), Milgrom and Webber (1982) and Bulow and Klemperer (1996) for details. Suppose the seller (target corporation) derives his own (reservation) value from the auctioned object of $v_0 \geq 0$. Recall that in a second price (Vickrey) auction, each bidder will reveal (bid) her true valuation ($b_i = v_i$), and thus we can confine ourselves to direct revelation mechanisms. Similarly, in an English auction, where there is a commonly observed ascending bid, it is each bidder's dominant strategy to stay in the auction until the ascending bid surpasses his or her valuation.

For each buyer-bidder i, let the random variable y represent the maximum of all remaining buyer's valuations, conditional on bidder i having the highest valuation. That is, $y = max\{v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_N\}$. Under the distributional assumptions above, it is easy to derive the conditional distribution function on y, which we denote as $G(y)$, is given by:

$$G(y) = F(y)^{N-1}$$

The density function is, then, given by:

$$g(y) = (N-1)F(y)^{N-2}f(y)$$

If there is no reservation price (r), then given the symmetry of the problem, each bidder with valuation v will expect to pay the following amount in the auction:

$$m(v) = \int_0^v y \cdot g(y)dy$$

In contrast, suppose a minimum reservation price of r is introduced, so that the winning bidder must make a bid larger than or equal to r. With the reservation price, the bidder expects to pay nothing if $v < r$, since she will not meet the minimum reservation price. In contrast, when $v \geq r$, the bidder of type v will expect to pay:

$$m(v; r) = r \cdot G(r) + \int_r^v y \cdot g(y)dy$$

The first expression represents the possibility that only the winning bidder has the valuation above the reservation price, in which case the winner will simply pay the reservation price.

The seller, of course, does not know the value of the bidder. But the seller can compute the expected payment of the bidder:

$$E_v\{m(v;r,N)\} = \int_r^1 \left(r \cdot G(r) + \int_r^v y \cdot g(y)dy \right) f(v)dv$$
$$= r \cdot G(r) \cdot [1 - F(r)] + \int_r^1 [1 - F(y)] \cdot y \cdot g(y)dy$$

(The second line requires changing the order of integration in the second term; it's a standard calculation but it is a bit tedious.) Consequently, the total payoff for the seller consists of the sum of the expected payments from each buyer plus the seller's own valuation the asset is retained. This payoff therefore is given by:

$$\Pi_s(v_0;r,N) = N \cdot E_v\{m(v;r,N)\} + F(r)^N v_0$$
$$= N \cdot \left(r \cdot G(r) \cdot [1 - F(r)] + \int_r^1 [1 - F(y)] \cdot y \cdot g(y)dy \right) + F(r)^N v_0$$

As an example, when there is no reserve price ($r = 0$) and the seller's reservation value is equal to zero ($v_0 = 0$), the seller's payoff simplifies considerably to be:

$$\Pi_s(v_0;0,N) = N \cdot \int_0^1 [1 - F(y)] \cdot y \cdot g(y)dy$$

What is the optimal reservation price (r) for a seller with reservation value of $v_0 \geq 0$? When we take the derivative of the seller's payoff respect to r, after some simplifications, we get:

$$\frac{\partial \Pi_s(v_0;r,N)}{\partial r} = N \cdot \left[1 - (r - v_0) \frac{f(r)}{1 - F(r)} \right] (1 - F(r)) \cdot F(r)^{N-1}$$

Note that this term is increasing in r when $r=0$, and thus for finite N, there is value to increasing the reserve price above zero.

Note that the second derivative is strictly negative for all interior r, so the interior root of the above equation must be a unique maximum. This unique maximum occurs at when the expression in the square brackets is zero, or:

$$r^* = \frac{1 - F(r^*)}{f(r^*)} + v_0$$

Note also that the optimal value of r is clearly greater than the seller's valuation v_0. In the case of a uniform distribution, where $f(v) = 1$ and $F(v) = v$, the above condition simplifies to:

$$r^* = \frac{1 + v_0}{2}$$

That is, the seller takes the average of her valuation and the top buyer valuation (of 1). In the context of our acquisition example, then, in the uniform case, the marginal shareholder would prefer a reserve price of $r^* = 1/2$, and both the pivotal and representative shareholders would favor a reserve price of $r^* = 3/4$.

It is interesting to note that the optimal reserve price does not hinge on the number of buyers in the auction. The maximized value for the shareholder is thus:

$$\Pi_s(v_0; r^*, N) = N \cdot \left[1 - (r - v_0)\frac{f(r)}{1 - F(r)}\right](1 - F(r)) \cdot G(r)$$

From the above derivations, a simple but helpful lemma follows:

Lemma A.1: *As N increases beyond* $N = 1$*, the marginal value of setting the reserve price r decreases for all values of* r*. In the limit, as N grows arbitrarily large, the marginal value of adjusting r approaches zero.*

Proof of Lemma A.1: First, note that for all r and $N > 1$,

$$\frac{\partial^2 \Pi_s(v_0; r, N)}{\partial r \partial N} = \left[1 - (r - v_0)\frac{f(r)}{1 - F(r)}\right]\big(1 - F(r)\big) \cdot F(r)^{N-1}(1 + N \cdot \ln{(F(r))} < 0$$

which is negative for $r < r^*$ and strictly positive for all $r > r^*$. Thus, the marginal benefit of moving *r* towards its optimal level attenuates as *N* grows beyond *N=1*. A simple application of L'Hopital's rule to the above expression confirms:

$$\lim_{N \to \infty} \frac{\partial^2 \Pi_s(v_0; r, N)}{\partial r \partial N} = 0$$

so that the in the limit a large number of bidders negates the value to the seller of setting *r*. ■

Appendix B: Sketches of Proofs

Proof of Proposition 1. The proof basically follows the analysis from Appendix A. Recall that the probability of sale, given by $\Pr\{Sale|N,r\} = 1 - F(r)^N$, strictly decreases as r increases. Therefore, the agent-manager, who wants to maximize $M \cdot \Pr\{Sale|N,r\} = M \cdot (1 - F(r)^N)$, will set $r = 0$.

To prove the last sentence of the Proposition, from Appendix A, we know that when $r = 0$, the expected sale price is given by:

$$E(v_{(N-1)}) = N(N-1) \int_r^1 [1 - F(y)] F(y)^{N-2} dF(y)$$

When $N = 1$, we get $E(v_{(N-1)}) = 0 < \mu$. ■

Proof of Proposition 2. Suppose the winning bid is given by b, and consider the effect of eliminating weakly dominated strategies for each shareholder type γ. Note that while there are infinitely many possible opponent strategy profiles ($\boldsymbol{\sigma}_{-\gamma}$) to consider, they fall into three categories: (1) $\boldsymbol{\sigma}_{-\gamma}$ have sufficiently many affirmative votes to approve the merger by a strict majority of votes; (2) $\boldsymbol{\sigma}_{-\gamma}$ have sufficiently many negative votes to veto the merger by a strict majority; and (3) $\boldsymbol{\sigma}_{-\gamma}$ results in a dead heat among other shareholders, so that the γ shareholder's vote is outcome-determinative.

Note that in scenarios (1) and (2), the shareholder's vote will not change the outcome, leaving the shareholder indifferent. This leaves scenario (3) in which approval hinges on shareholder's vote. Here, shareholder γ would strictly prefer to vote against the merger when $\gamma < b$ and at least weak preference to vote for it when $\gamma \geq b$. (We assume without loss of generality that any tie is broken in favor of the merger). Consequently, sincere voting is weakly dominant for all these types. This immediately implies that the acquisition will be approved if and only if it is attractive to the pivotal voter ($b \geq \rho$). Anticipating this effect, the lowest reserve price the agent-manager can credibly announce is $r_m = \rho$. The target-shareholder payoff stated in the Proposition thus corresponds to the expected seller's payoff from an ascending-bid English auction when the reservation price fixed at ρ. ■

Proof of Proposition 3. Let $b \in [0,1]$ denote the winning bid and let the appraisal price $\phi(b)$ reflect the MP rule, so that $\phi(b) = b$. Conditional on the merger taking place, all shareholders are indifferent between accepting the terms of the merger or seeking appraisal. Regardless of their decision, they receive b, and again (without loss of generality) we will assume that indifferent shareholders opt to take the terms of the merger. Appraisal rights under the MP thereby have no bearing on the agent's decision choice of a reserve price. And, since the agent's payoff is strictly decreasing in r, the manager once again sets $r_m = 0$. Facing a zero reserve price, all bidders participate. As with Proposition 1, he firm will be sold with probability one at price equal to $v_{(N-1)}$. ■

Proof of Proposition 4. Suppose that the agent agrees to a merger at price *b*. With no shareholder voting, the merger is sure to occur. For each shareholder, the strategy choice boils down to choosing between taking the merger consideration b or seeking appraisal at a "fair value" equal to ϕ.

Suppose first that the winning bid is such that $b < \phi$. Conditional on there being a merger, for all shareholders, the dominant strategy is to seek appraisal and receive ϕ. If the winning bid is such that $b > \phi$, on the other hand, the strictly dominant strategy for all shareholders is to eschew appraisal and receive b. If the winning bid is such that $b = \phi$, conditional on there being a merger, all shareholders are indifferent between accepting the merger terms or seeking appraisal. We will assume once again that the shareholder will accept the merger consideration.

Given the shareholders' strategies, for each bidder, if she wins the auction with a bid of $b < \phi$, the bidder will must nonetheless pay ϕ instead of b for the company. On the other hand, if she wins the auction with a bid of $b \geq \phi$, she will pay b for the company. Consequently, any bidder whose valuation is such that $v < \phi$ will immediately drop out. On the other hand, if her valuation is (weakly) higher than ϕ, she will stay in the auction until the ascending bid reaches her valuation. This is equivalent to an auction in which the reserve price is set at $r_m = \phi$.

In equilibrium, if $v_{(N)} < \phi$, which occurs with probability $F(\phi)^N$, the firm will not be sold. On the other hand, if $v_{(N)} \geq \phi$, which occurs with probability $1 - F(\phi)^N$, the firm will be sold at merger price equal to $\max\{\phi, v_{(N-1)}\}$. The ordering of shareholder welfare follows naturally from the definition of r^*. ■

Proof of Proposition 5. Suppose $\phi < \rho$. We will divide the proof into three cases: (1) $b < \phi < \rho$; (2) $\phi \leq b < \rho$; and (3) $\phi < \rho \leq b$.

Case 1: suppose that the winning bid is such that $b < \phi < \rho$. Consider shareholder type $\gamma \in (\phi, 1]$, and three distinct scenarios: (1) merger takes place regardless of the shareholder's vote; (2) merger fails regardless of the shareholder's vote; and (3) the shareholder's vote is outcome determinative. In the first scenario, the shareholder's dominant strategy is to vote against the merger and exercise appraisal (since $b < \phi$). In the second, the shareholder is indifferent among strategies. In the third, the shareholder's dominant strategy is to vote against the merger to prevent it from occurring. Hence, for all shareholders with $\gamma \in (\phi, 1]$, the weakly dominant strategy is to vote sincerely against the merger and seek appraisal. The merger thus fails. Conditional on the merger failing, the other shareholders' votes become irrelevant. (Weak dominance does not, in contrast, constrain the strategies of shareholders with values less than ϕ, and thus there is not a unique undominated equilibrium for these voters. However, their votes are never sufficient to tip the balance on the merger.)

Case 2: suppose now that $\phi \leq b < \rho$. Because $b \geq \phi$, no dissenting shareholders will seek appraisal. Consider shareholder type $\gamma \in (b, 1]$, and the three possibilities. If merger will take place regardless of the shareholder's vote, the shareholder is indifferent about her vote. If merger does not take place regardless of the shareholder's vote, the shareholder is also indifferent among different strategies. If the shareholder's vote is outcome determinative, the shareholder's dominant strategy is to vote against the merger, since $\gamma > b$. The shareholder's weakly dominant strategy, therefore, is to vote against the merger and seek no appraisal in case the merger takes place. The bid does not get enough votes from the shareholders, and the merger fails.

Case 3: suppose $\phi < \rho \leq b$. Once again, since $b \geq \phi$, no dissenting shareholder will exercise appraisal. Consider shareholder type $\gamma \in [0, \rho]$. If the merger were to take place regardless of the shareholder's vote, the shareholder is indifferent about her vote. If the merger were not to take place regardless of the shareholder's vote, the shareholder is indifferent among different strategies. Finally, if the shareholder's vote is outcome determinative, the dominant strategy is to vote for the merger to receive $b \geq \gamma$. The weakly dominant strategy for the shareholder with type $\gamma \in [0, \rho]$ is therefore to vote for the merger; the merger thus succeeds. All shareholders receive the consideration of b and no one exercises the appraisal remedy. ■

Proof of Lemma 6A. Suppose $\phi < \rho$. We will divide the proof into three cases: (1) $b < \phi < \rho$; (2) $\phi \leq b < \rho$; and (3) $\phi < \rho \leq b$.

Case 1: suppose that the winning bid is such that $b < \phi < \rho$. Consider shareholder type $\gamma \in (\phi, 1]$. Consider three scenarios: (1) merger takes place regardless of the shareholder's vote; (2) merger fails regardless of the shareholder's vote; and (3) the shareholder's vote is outcome determinative. In the first scenario, the shareholder's dominant strategy is to vote against the merger and exercise appraisal. In the second, the shareholder is indifferent across different strategies. In the third, the shareholder's dominant strategy is to vote against the merger and seek appraisal. Hence, for all shareholders with $\gamma \in (\phi, 1]$, the weakly dominant strategy is to vote against the merger and seek appraisal. The merger fails. Conditional on the merger failing, the other shareholders' votes become irrelevant.

Case 2: suppose $\phi \leq b < \rho$. Because $b \geq \phi$, no dissenting shareholders will seek appraisal. Consider shareholder type $\gamma \in (b, 1]$, and the three possibilities. If merger takes place regardless of the shareholder's vote, the dominant strategy for the shareholder is to vote against and not exercise appraisal. If merger does not take place regardless of the shareholder's vote, the shareholder is indifferent among different strategies. If the shareholder's vote is outcome determinative, the shareholder's dominant strategy is to vote against the merger, since $\gamma > b$. The shareholder's weakly dominant strategy, therefore, is to vote against the merger and seek no appraisal in case the merger takes place. The bid does not get enough votes from the shareholders and the merger fails.

Case 3: suppose $\phi < \rho \leq b$. Again, since $b \geq \phi$, no dissenting shareholder will exercise appraisal. Consider shareholder type $\gamma \in [0, \rho]$. If the merger were to take place regardless of the shareholder's vote, the dominant strategy is for the shareholder to vote for the merger. If the merger were not to take place regardless of the shareholder's vote, the shareholder is indifferent among different strategies. Finally, if the shareholder's vote is outcome determinative, the dominant strategy is to vote for the merger to receive $b \geq \gamma$. The weakly dominant strategy for the shareholder with type $\gamma \in [0, \rho]$ is to vote for the merger and the merger will succeed. All shareholders receive the consideration of b and no one exercises the appraisal remedy. ■

Proof of Lemma 6B.1. Suppose $\rho \leq \phi$. Let's consider two separate cases: (1) $b < \rho \leq \phi$ and (2) $\rho \leq \phi \leq b$.

Case 1: $b < \rho \leq \phi$. Because $b < \phi$, all dissenting shareholders will seek appraisal. Consider shareholder type $\gamma \in (b, 1]$. If the merger were to pass regardless of the shareholder's vote, the dominant strategy is for the shareholder to vote against the merger and seek appraisal. If the merger were to fail, the shareholder is indifferent across strategies. Finally, if the shareholder is pivotal, the dominant strategy is to vote against the merger since $\gamma > b$. The weakly dominant strategy for the shareholders is to vote against the merger and seek appraisal remedy. Merger will fail since $b < \rho$.

Case 2: $\rho \leq \phi \leq b$. Since $\phi \leq b$, no dissenting shareholder will seek appraisal. Consider shareholder type $\gamma \in [0, b]$. First, if the merger were to take place regardless of that shareholder's vote, the dominant strategy is for the shareholder to vote for the merger. Second, if the merger is to fail regardless of the shareholder's vote, the shareholder is indifferent among different strategies. Third, if the shareholder's vote is outcome determinative, the dominant strategy is to vote for the merger. Hence, the weakly dominant strategy for the shareholder is to vote for the merger. Since $\rho \leq b$, merger succeeds and all shareholders receive the merger consideration. ■

Proof of Lemma 6B.2. Suppose that $\rho \leq b < \phi$, so that the bid price is attractive to the pivotal shareholder, but the appraisal value is even more attractive. Whenever a merger is conjectured certain to occur, all shareholders would prefer to vote against and seek appraisal. That outcome is clearly not attainable, since 50% of the shareholders must vote for it. Thus, any equilibria involving the approval of the merger can never have more than a bare majority supporting the sale—thereby making every affirmative vote pivotal. The key question is how the account of "no" votes is allocated among the shareholders in this case.

Consider first the group of shareholders for whom $\gamma > \phi$, and thus they are not in favor of the merger even with the appraisal option. For these shareholders, it is weakly dominant to vote against the merger all the time, since they do not want the deal to go through (even if they are pivotal), and they would rather receive the appraisal if it does occur. A similar (but slightly more nuanced) form of reasoning applies to shareholders for whom $\gamma \in (b, \phi]$; these types have valuations exceeding the bid b but less than the appraisal value ϕ. They would most prefer to see the merger approved, but seek appraisal—which requires them to vote against. If that is not possible, their next best favored outcome is that the merger is not approved, in which case they could enjoy their status quo payoffs—which also prescribes voting against. Their least preferred scenario is to vote for a merger where they vote in favor. Thus, this group will vote against the merger as well. We thus know that all shareholders with $\gamma > b$ will vote sincerely against the merger.

Now consider the set of shareholders for whom $\gamma \leq b$, which consists of the union of shareholders for whom $\gamma \in (\rho, b]$ as well as those for whom $\gamma \in [0, \rho]$. Both groups have the same set of payoffs here: both would support the merger on its own terms, since it offers more than their valuation. But at the same time, both groups would prefer to seek appraisal if they knew the deal was going to be approved. This poses a significant coordination problem. If there were ever an equilibrium where all of these shareholders voted for the merger, then some would have a strict incentive to defect and vote against in order to perfect appraisal rights. Indeed, there cannot be an equilibrium involving approval unless votes in favor marshal a bare minimum 50% vote. There are infinite ways to marshal this vote, but in all of them, the relatively low-valuing shareholders (for whom $\gamma \leq b$) must coordinate on a way to ration their strategic no votes so as to preserve the approval of the merger, assuming all others vote against. In contrast, there are a continuum of equilibria in which these low-valuing shareholders overwhelmingly vote no. Both types of equilibria are robust to the elimination of weakly dominated strategies. ■

Proof of Proposition 6. Part (A) follows from Lemma 6A where ρ becomes the de facto reserve price. The manager maximizes her expected return by setting $r = \rho$. Part (B)(1) follows from Lemmas 6B.1 and 6B.2. ■

Proof of Proposition 7. Recall first, from Part II.A, that $r^* > \mu$. When $\rho > r^*$, because the pivotal shareholder's valuation exceeds the optimal reserve price, per Propositions 5 and 6, setting $\phi > \rho$ will only reduce the target shareholders' expected return. Hence, the optimal appraisal rule is to either use the MP rule or to set $\phi \leq \rho$.

When $\rho \leq r^*$, voting threshold is insufficiently high to maximize the target shareholders' return. The optimal appraisal rule depends on the type of equilibrium obtained. Per Proposition 6(B)(1), in the uncoordinated equilibrium, ϕ becomes the de facto reserve price. The optimal appraisal rule, therefore, is to set $\phi = r^*$. In the coordinated equilibrium, per Proposition 6(B)(2), with $\rho = H^{-1}(\alpha)$, α fraction of the shareholders receive b while the remaining shareholders ($1 - \alpha$ fraction) get ϕ. The expected payment by the winning bidder is $\alpha b + (1 - \alpha)\phi$. The minimum winning bid is equal to ρ, in which case the buyer's expected payment is $\alpha\rho + (1 - \alpha)\phi$. To create the de facto reserve price of r^*, the court needs to set ϕ such that $\alpha\rho + (1 - \alpha)\phi = r^*$ or, equivalently, $\phi = \frac{r^* - \alpha\rho}{1 - \alpha}$, subject to the constraint of $\frac{r^* - \alpha\rho}{1 - \alpha} \leq 1$. The optimal appraisal rule, therefore, is: $\phi = \min\left\{1, \frac{r^* - \alpha\rho}{1 - \alpha}\right\}$. ■

References

Allen, William T., "Securities Markets as Social Products: The Pretty Efficient Capital Market Hypothesis," J. Corp. L. 28:551-63 (2003).

Ausubel, Lawrence & Peter Cramton. "Vickrey Auctions with Reserve Pricing." *Economic Theory* 23:493—505 (2004).

Bainbridge, Stephen. "Elements of Value Arising from the Accomplishment or Expectation of the Merger." https://www.professorbainbridge.com, posted 5/25/2012.

Batina, Raymond G. & Toshihiro Ihori. *Public Goods: Theories and Evidence*, Springer Verlag, (2005).

Bomba, Abigail, et. al. "Why Delaware Appraisal Awards Exceed the Merger Price—The Gap between What the Delaware Chancery Court Says and Does." https://corpgov.law.harvard.edu, posted 9/23/2014.

Brunnermeier, Marcus, Alp Simsek, & Wei Xiong. "A Welfare Criterion for Models with Distorted Beliefs." *Quarterly Journal of Economics* 129:1753—1797 (2014).

Bulow, Jeremy & Paul Klemperer. "Auctions Versus Negotiations." *American Economic Review* 86(1):180–94 (1996).

Cai, Hongbin, John Riley, & Lixin Ye. "Reserve Price Signaling." *Journal of Economic Theory* 135:253—268 (2007).

Choi, Albert & Eric Talley. "Tender Offer versus Statutory Merger." Unpublished working paper (2016).

Cornell, Bradford, "The Incentive to Sue: An Option-Pricing Approach," *J. Legal Stud.* 19:173-87 (1990).

Duggan, John. "A Note on Backward Induction, Iterative Elimination of Weakly Dominated Strategies, and Voting in Binary Agendas." Unpublished working paper (2003).

Gelman, Andrew, Jonathan Katz, and Gary King. "Empirically Evaluating the Electoral College," in *Rethinking the Vote: The Politics and Prospects of American Electoral Reform* (Crigler, et. Al. eds., 2003).

Hamermesh, Lawrence & Michael Wachter. "The Fair Value of Cornfields in Delaware Appraisal Law." *The Journal of Corporation Law* 31:119-166 (2005).

Hamermesh, Lawrence & Michael Wachter. "The Short and Puzzling Life of the Implicit Minority Discount in Delaware Appraisal Law." *University of Pennsylvania Law Review* 156:1-61.

Korsmo, Charles & Minor Myers. "Appraisal Arbitrage and the Future of Public M&A." *Washington University Law Review* 92:1551-1615 (2015).

Korsmo, Charles & Minor Myers. "Reforming Modern Appraisal Litigation," 41 *Delaware Journal of Corporate Law* forthcoming (2016).

Krishna, Vijay. *Auction Theory*, Academic Press, 2002.

Levmore, Saul & Hideki Kanda, "The Appraisal Remedy and the Goals of Corporate Law." UCLA Law Review 32:429-73 (1985).

Milgrom, Paul & Robert Webber. "A Theory of Auctions and Competitive Bidding," *Econometrica* 50:1089-1122 (1982).

Myerson, Roger B. "Optimal Auction Design." *Mathematics of Operations Research* 6(1):58-73 (1981).

Patty, John W., James M. Snyder, Jr., & Michael M. Ting. "Two's Company, Three's an Equilibrium: Strategic Voting and Multicandidate Elections." *Quarterly Journal of Political Science* 4:251–278 (2009).

Stulz, Rene. "Managerial Control of Voting Rights: Financing Policies and the Market for Corporate Control." *Journal of Financial Economics* 20:25—54 (1988).

Talley, Eric. "Expectations and Legal Doctrine," *in* Paradoxes and Inconsistencies in the Law 183-204 (O. Perez & G. Taubner, eds. 2006).

Thompson, Robert B. "Exit, Liquidity, and Majority Rule: Appraisal's Role in Corporate Law." *Georgetown L. J.* 84:1-54 (1995)